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Innovative Approaches to promote Inclusive and Quality Mathematics Education in Africa: Case Studies from Kenya and South Sudan

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Abstract

This master thesis delves into innovative strategies for advancing inclusive and quality mathematics education in Africa, with a focus on case studies from Kenya and South Sudan. The thesis contextualizes the importance of mathematics education in the SDG era, highlighting its critical role in achieving the 2030 Sustainable Development Agenda. To address the research question of how to facilitate the attainment of SDG 4 educational goals, particularly in sub-Saharan Africa, the thesis explores the potential of innovative approaches such as games, technology, project-based learning, and ethnomathematics.
The case study on Kenya provides a comprehensive overview of mathematics initiatives in the country, detailing the author's experience organizing and executing a math camp aimed at improving students' maths skills and fostering their interest in science. The Kenya Math Camp is thoroughly described, including its objectives, structure, planning, and evaluation, with a mathematical activity, the Josephus Problem, as an example.
Similarly, the case study on South Sudan provides insights into the author's experience teaching mathematics in the country and its effectiveness in improving mathematical skills. The author explains the background and methodology of the teaching, evaluation, and feedback received, with an ethnomathematics activity as an example.
In conclusion, the thesis underscores the urgent need for innovative approaches and initiatives to promote mathematics education in Africa, with the case studies from Kenya and South Sudan demonstrating the effectiveness of game-based learning, technology integration, project-based learning, and ethnomathematics in enhancing students' engagement and understanding. This research aims to contribute to the growing body of knowledge on improving mathematics education and highlights the transformative power of education in driving sustainable development.

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Monica

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Mathematics may not teach us how to add love or subtract hate, but it gives us hope that every problem has a solution.

Dr. Mae Jemison, first African American woman to travel in space.

Introduction

The 21st century has brought with it several challenges, including climate change, mass migrations, and increasing disparities across the globe. The impacts of these challenges are felt in every corner of the world, affecting both developed and developing countries. Climate change, in particular, poses an existential threat to the future of humanity, as rising temperatures, sea levels, and extreme weather events become more frequent and severe. At the same time, mass migrations have become a defining feature of the modern era, as millions of people are displaced from their homes due to conflict, economic hardship, and environmental degradation. These migrations often intensify existing inequalities, as those with fewer resources and opportunities are more likely to be forced to flee. Meanwhile, the gap between the rich and poor continues to widen, both within and between countries, as economic and political power becomes increasingly concentrated in the hands of a privileged few. Addressing these challenges will require coordinated, sustained efforts from governments, civil society organizations, and individuals alike.

The urgency for action is what forms the basis of this thesis. Specifically, it aims to address the urgent need to improve the state of education in sub-Saharan Africa and contribute to achieving the goals of the 2030 Agenda, specifically Sustainable Development Goal 4 (SDG 4), which aims to ensure that all children have access to quality education and lifelong learning opportunities. Investing in education in sub-Saharan Africa is crucial for addressing global challenges, as it can enhance the capacity of individuals and societies to adapt and mitigate the impacts of climate change, reduce the drivers of migration by providing better opportunities and reducing poverty, and decrease global disparities by fostering economic growth and development. Additionally, improving education outcomes in sub-Saharan Africa can have a positive ripple effect on other regions of the world, particularly for countries that receive significant migration flows from the region, such as Europe. By creating a more skilled and knowledgeable community in sub-Saharan Africa, we can improve the potential contributions of individuals who migrate to other countries, ultimately benefiting the global community as a whole.

To accelerate progress towards achieving the SDG 4, this thesis proposes two mathematics initiatives, one in Kenya and one in South Sudan. The initiatives are designed to address the existing gaps in mathematics education by creating innovative didactic resources, providing comprehensive teacher training, and collaborating with local schools and institutions. Through these initiatives, the aim is to not only improve the quality of mathematics education but also enhance the skills of both students and teachers, enabling them to better contribute to their local communities and promote sustainable development. By leveraging technology and gamification, the initiatives aim to make math more engaging and accessible to students, particularly those from marginalized backgrounds. Furthermore, by focusing on ethnomathematics and exploring the cultural and historical roots of mathematics, the initiatives aim to make mathematics education more culturally relevant and inclusive. The proposed initiatives represent a promising step towards creating a more effective and sustainable education system that empowers learners and educators to thrive in the 21st century.

The master's thesis is structured to provide a thorough analysis of mathematics education in the context of the SDG era, with a specific focus on two African countries, Kenya and South Sudan. The introduction section provides an overview of the topic and its relevance to the SDG agenda. The first chapter delves into the 2030 Sustainable Development Agenda and international agreements related to education, with a particular emphasis on SDG 4. The main focus is on the role and challenges of mathematics in education, particularly in low-income countries. Five key themes, essential for the improvement of mathematics education, are identified: access, equity, learning, quality, and finance. To provide a comprehensive understanding of the current state of education worldwide, the chapter presents a global overview of each theme. By doing so, it aims to contribute to the ongoing discussion on how to achieve sustainable and equitable education for all. The subsequent section includes an in-depth review of relevant literature on the use of games, technology, project-based learning, and ethnomathematics, and an analysis of their effectiveness in promoting mathematical engagement and achievement among students, providing valuable insights into their potential benefits and limitations. By exploring the latest research in this field, this section aims to provide a comprehensive overview of potential strategies for improving the effectiveness of mathematics education in the SDG era. The second chapter of the thesis provides an in-depth exploration of the Kenya Math Camp, which is a unique initiative that aims to enhance critical thinking, problem-solving skills, and creativity among students while fostering a love for mathematics. This chapter includes a detailed analysis of the objectives, structure, and impact of the camp. The Josephus Problem, which is a classic mathematical problem, is presented as an example of the mathematical activities offered at the camp. Through this activity, students learn how to apply mathematical concepts to real-world situations and develop problem-solving skills. The chapter also analyzes the effectiveness of the initiative in promoting a deeper understanding of mathematics among students and highlights the key takeaways for future implementations of similar initiatives in other low-income countries. The third chapter of the thesis delves into the efforts aimed at enhancing mathematics education in South Sudan, with a particular focus on the use of ethnomathematics. This chapter provides a detailed overview of the current state of mathematics education in South Sudan, while highlighting the challenges that the country is facing. In addition, it outlines the background and methodology of the teaching approach, which includes an evaluation and feedback process. A mathematical activity, "From Fractals to Complex Numbers", is presented as an illustrative example of the approach. This activity utilizes culturally relevant and context-specific mathematical examples to improve the students' learning experience. The chapter further presents a comprehensive session-by-session breakdown of the initiative's structure, covering the objectives, methodology, and evaluation process. The thesis concludes with a section on future developments and potential applications of the teaching approach discussed in the third chapter. In particular, the successful use of games, technology, and project-based learning, as well as the incorporation of etnomathematics in mathematical activities, can inform and benefit mathematics education in Europe, where educators face challenges of promoting inclusion and diversity in the classroom, due to the presence of students from diverse backgrounds and cultures.

It is important to acknowledge the limitations and challenges encountered during this research. Due to time and resource constraints, the scope of the study was limited to two countries in sub-Saharan Africa, Kenya and South Sudan. Additionally, the research relies on feedback and comments from teachers and students, which may be subject to bias and limitations in the accuracy and completeness of the responses. Furthermore, the initiatives were implemented in a relatively short period, which may limit the ability to assess their long-term impact. Despite these limitations, this thesis provides valuable insights and evidence-based for improving mathematics education in sub-Saharan Africa. The findings of this thesis have significant implications for future research and policy decisions in the field of education and sustainable development in sub-Saharan Africa. By identifying successful strategies for improving mathematics education in the region, this study can inform the development of more effective and sustainable education interventions that can enhance the capacity of individuals and societies to address global challenges. Moreover, the research highlights the importance of investing in education in sub-Saharan Africa, which can contribute to reducing poverty and inequality, mitigating the impacts of climate change, and reducing the drivers of migration. By investing in education in African countries, this thesis hopes to contribute to the larger goal of achieving sustainable development worldwide, which Our Common Future (1987) defines as "development that satisfies the needs of the present without compromising the ability of future generations to meet their own needs". By investing in education in sub-Saharan Africa, the end goal is indeed to ensure that future generations are better equipped to meet their needs and achieve sustainable development.

Chapter 1

Mathematics Education in the SDG Era

Education is a crucial aspect of sustainable development and a fundamental human right recognized by the United Nations. The 2030 Sustainable Development Agenda has set out to achieve Sustainable Development Goal 4 (SDG 4), which aims to ensure that all children have access to quality education and lifelong learning opportunities. This goal is especially important in light of the challenges facing the 21st century, such as climate change, migrations, and disparities around the world.

While progress has been made towards achieving the Goal, significant challenges persist. The COVID-19 pandemic has further exposed and amplified these challenges, particularly in low-income countries where access to education is already limited. Moreover, the pandemic has widened the digital divide, exacerbating the difficulty many students face in accessing online learning.

Mathematics is not only a core subject in most education systems but also a critical component of sustainable development. However, mathematics education faces several challenges, including a high number of students who struggle with math, resulting in low achievement and disengagement from the subject. Additionally, access to quality education is limited in low-income countries, making it difficult for students to receive adequate mathematics instruction. This can have long-term consequences, as mathematics is essential for many higher education programs and careers.

This chapter aims to provide a comprehensive overview of the current state of education and mathematics education within the framework of the 2030 Sustainable Development Agenda. It explores the progress made towards achieving SDG 4, the role of mathematics in sustainable development, and the challenges facing mathematics education in particular in low-income countries. Finally, two different approaches will be presented as potential solutions to these challenges. The first approach focuses on improving student motivation and engagement through the use of games, technology, and project-based learning. The second approach explores the relationship between culture and mathematics and how it can be utilized in the classroom context.

1.1 The 2030 Sustainable Development Agenda

The 2030 Sustainable Development Agenda is a 15-year long path, established in 2015 and shared by all the UN Member States, focusing on the actions needed to bring peace and prosperity to everyone, everywhere. Ending poverty and other deprivations, while improving health and education, reducing inequalities, and stimulating economic growth, tackling climate change and working to preserve oceans and forests, must be key values acknowledged by every country, developed, and developing. As part of the 2030 Agenda for Sustainable Development, 17 Sustainable Development Goals (SDGs) have been established as a universal call to action by all countries. The 17 SDGs and the 169 targets demonstrate the ambition of the Agenda and revolve around the three dimensions for sustainable development: economic, social and environmental. There are 5 macro-areas on which to focus attention:

1. People: the main objective is to empower every human being, helping with the fulfillment of their potential, surrounding them by dignity, equality and a healthy environment. Ending poverty and hunger is the first step to advance this goal.

2. Planet: it must be protected by degradation, promoting sustainable productions and consumptions, an improvement management of natural resources and a top-priority call against climate change. The planet must be safeguarded for the future generations.

3. Prosperity: every human being should be able to live a prosperous and fulfilling life, which moves in parallel with economic, social and technological progress in harmony with nature and the planet.

4. Peace: the entanglement between sustainable development and peace is inseparable, one cannot exist without the other. The Agenda is committed to promote inclusive societies, free from violence and fear, just-based and peaceful.

5. Cooperation: the member states undertake to renew the partnership for sustainable development, particularly focused on the needs of the poorest and most vulnerable, through a spirit of global solidarity.

History of Agenda

Decades of cooperation between Countries, the United Nations and the ONU Department of Economic and Social Affairs – DESA brought to light the 2030 Agenda:

- June 1992, Earth Summit, Rio de Janeiro, Brazil. On this occasion, more than 178 countries adopted the Agenda 21. This was a global action plan to build a global partnership, with the goal of safeguarding the environment and improving the life of every human being.

- September 2000, Millenium Summit, New York, US. The member states unanimously adopted the United Nations Millennium Declaration. The summit ended with the formulation of 8 Millenium Development Goal (MDG) to reduce extreme poverty within 2015.

- August-September 2002, World Summit on Sustainable Development, Johannesburg, South Africa. In this occasion, the Johannesburg Declaration on Sustainable Development and Plan of Implementation were adopted to renew the global community's commitments to poverty eradication and environmental sustainability. These agreements built upon previous international commitments such as Agenda 21 and the Millennium Declaration, with a stronger focus on the importance of multilateral partnerships to achieve sustainable development goals.

- June 2012, United Nations Conference on Sustainable Development (Rio+20), Rio de Janeiro, Brazil. At this event, member states adopted the document "The Future We Want" and initiated a process to develop a set of Sustainable Development Goals based on the Millennium Development Goals. In addition, the United Nations High-Level Political Forum on Sustainable Development was established.

- Later, in 2013, the General Assembly established a working group to develop a proposal of Sustainable Development Goals.

- January 2015, the General Assembly launched the negotiation process on the post-2015 development agenda. The process climaxed in the subsequent adoption of the 2030 Agenda for Sustainable Development, with 17 SDGs at its core (the United Nations Summit on Sustainable Development, September 2015).

- 2015 was a landmark year for multilateralism and international policy-making, with the adoption of several important agreements:
  - Sendai Framework for Disaster Risk Reduction (March 2015)
  - Addis Agenda on Financing for Development (July 2015)
  - United Nations Sustainable Development Summit 2015: 2030 Sustainable Development Agenda and the 17 SDGs (September 2015)
  - Paris Agreement on Climate Change (December 2015)

The 17 Soustainable Development Goals.

The 17 Sustainable Development Goals (SDGs) were adopted by the United Nations General Assembly in 2015 as part of the 2030 Agenda for Sustainable Development. These goals serve as a blueprint for the international community to achieve a better and more sustainable future for all. The SDGs cover a wide range of interconnected themes including eradicating poverty, promoting good health and well-being, ensuring quality education, achieving gender equality, providing access to clean water and sanitation, promoting sustainable economic growth, reducing inequality, and combating climate change, among others.
Each SDG has specific targets and indicators that help measure progress toward the goal. For example, SDG 1 aims to ensure that all people have access to basic resources and

services, including food, water, healthcare, and education, while SDG 5 seeks to eliminate all forms of discrimination and violence against women and girls and to promote women's full and effective participation in all areas of life. Targets for SDG 10 include reducing inequalities within and among countries, promoting social, economic, and political inclusion for all, and ensuring equal opportunities for all regardless of their background. It is true that each SDG is unique and addresses a different aspect of sustainable development, but they are all interconnected and interdependent. For instance, achieving gender equality (SDG 5) is crucial for reducing inequalities (SDG 10) since gender discrimination is one of the major drivers of inequality globally. Discrimination against women and girls in access to education, health care, and employment opportunities perpetuates a cycle of poverty and exclusion, which in turn fuels wider societal inequalities. Similarly, addressing inequalities in income, social status, and access to resources can help reduce gender discrimination and promote a more equal and just society for all individuals, regardless of gender.

Figure 1.1: Sustainable Development Goals

Achieving the SDGs requires a collective effort from all member states, who have committed to taking specific actions to meet the goals by 2030. The member states are expected to integrate the SDGs into their national development plans, policies, and strategies, as well as to mobilize resources, implement effective monitoring and evaluation systems, and ensure participation and ownership by all stakeholders, including civil society, private sector, and academia. Additionally, the member states are encouraged to share knowledge, best practices, and lessons learned with one another, and to collaborate and cooperate at regional and international levels. The SDGs represent an ambitious and transformative agenda for sustainable development, and their successful implementation will require sustained political will, strong leadership, and effective partnerships between all stakeholders.

In this master's thesis, our focus will be on Sustainable Development Goal 4 and its associated targets, which aim to tackle the challenges facing the education system. As such, the following section will outline the agreements that led to the subscription of the Sustainable Development Goal 4 and provide a comprehensive overview of the current global situation regarding education.

1.2 International Agreements and Education

For over thirty years, international agreements aimed at promoting sustainable development have recognized education as a fundamental aspect. The World Declaration on Education for All was adopted in 1990 during the World Conference on Education in Jomtien, Thailand. During this conference, delegates from 155 countries and representatives of around 150 governmental and non-governmental organizations agreed to significantly decrease illiteracy by making primary education accessible to everyone by the end of the decade. The Declaration focused on several goals, including universal access to learning, fairness, learning outcomes, expansion of basic education, improvement of the learning environment, and strengthening partnerships. The Declaration emphasized literacy as a fundamental human right and urged all countries to make efforts to meet the basic learning needs of all by the year 2000, UNESCO (1990).

The United Nations Conference on Environment and Development, held in Rio de Janeiro in 1992, also highlighted the crucial role of education in promoting sustainable development. Paragraph 3 of Chapter 36 of Agenda 21 emphasized that education is critical for promoting sustainable development and improving people's capacity to address environmental and development issues. Basic education provides the foundation for environmental and development education, which needs to be incorporated as an essential part of learning, United Nations (1992). Eight years later, the Dakar Framework for Action was adopted during the World Education Forum Education for All: Meeting our Global Commitments in Dakar in April 2000. This was the first international opportunity to observe and evaluate the results achieved by the Education for All initiative. The evaluation produced a detailed analysis of the state of basic education in the world, and each country assessed its own progress toward the Jomtien targets and reported the results at six regional conferences held in 1999 and 2000. The six regional EFA frameworks adopted at these conferences are an integral part of the Framework for Action, UNESCO (2000).

In September 2000, 191 heads of state and government signed the United Nations Millennium Declaration, which contains eight milestones to be achieved by 2015. The Millennium Development Goal 2 aims to achieve universal primary education and ensure that children everywhere, boys and girls alike, will be able to complete a full course of primary schooling, United Nations (2000). The World Summit on Sustainable Development adopted the Johannesburg Plan of Implementation in 2002, which reiterated both Millennium Development Goal 2 and the Dakar Framework for Action target on education in Section X of the Plan. The goal was to eliminate gender disparity in primary and secondary education by 2005 and at all levels by 2015.

At the UNESCO World Conference on Education for Sustainable Development in November 2014, the final report of the United Nations Decade for Education for Sustainable Development *Shaping the Future We Want* was released, UNESCO (2014). Member states reaffirmed their commitment to the right to education and pledged to strengthen international cooperation to achieve universal access to primary education, especially for developing countries. They emphasized that achieving full access to quality education at all levels is a prerequisite for sustainable development, poverty eradication, gender equality, women's empowerment, and human development. The report also highlighted the importance of equal access to education for everyone, including people with disabilities, indigenous peoples, local communities, ethnic minorities, and people living in rural areas.

In May 2014, an international meeting was held in Muscat, Oman to discuss Education for All. During this meeting, the Muscat Agreement was established, which identified education as a crucial goal for global development from 2015 to 2030. As a result, participants committed to mobilizing international support for the overarching objective of ensuring equitable and inclusive quality education and lifelong learning for all by 2030.

Given the international commitment since 1990 to promote education as a critical component of sustainable development, it is unsurprising that one of the 17 Sustainable Development Goals (SDGs) outlined in the 2030 Agenda focuses specifically on quality education with a view to promoting equity and inclusivity.

1.2.1 SDG 4: an Overview of Global Situation

The SDG 4 of the 2030 Agenda states "Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all", see 1.2. It aims to assure a basic level of education for all children and youths, providing an easy access to learning facilities and other learning opportunities. SDG 4 is composed by 10 targets, seven of which are "outcome-oriented targets" and the rest are "means of achieving targets". The former are: free primary and secondary education (4.1); equal access to quality pre-primary education (4.2); affordable technical, vocational and higher education (4.3); increased number of people with relevant skills for financial success (4.4); elimination of all discrimination in education (4.5); universal literacy and numeracy (4.6); and education for sustainable development and global citizenship (4.7). The latter three strategies are: build and upgrade inclusive and safe schools (4.a); expand higher education scholarships for developing countries (4.b); and increase the supply of qualified teachers in developing countries (4.c).

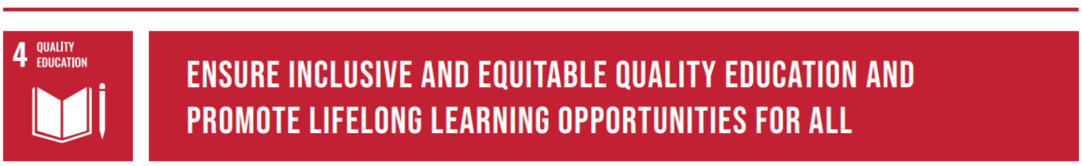


Figure 1.2: Sustainable Development Goal 4

Analysing the ten targets of SDG 4, five key themes can be identified: access, equity, learning, quality and finance. In the following, a global overview of each theme will be presented to highlight the current state of education in the world.

Access

Globally, 87% of children complete primary school, 77% of adolescents lower secondary school and 58% of young people upper secondary school (UNESCO, 2020). Over the last years, these rates have gradually increased, showing a positive trend in school completion around the world. However, a big difference is present when looking at the different rates according to the region's income. In fact, the lower secondary school completion rate in 2020 reached 97% for high-income countries, remaining at 35% for low-income countries, with the example of Guinea Bissau having a rate of 13.4%. This huge difference can also be observed geographically around the world, highlighting areas where the lack of education is specific to a larger territory, e.g. sub-Saharan Africa, Figure 1.3.
Sustainable Development Goal 4 aims for all young people in the world to complete secondary school by 2030, which means that every child should be enrolled in the school system by 2018. In reality, only 70 per cent of children in low-income countries have started the process, which means that a large number of children will still remain excluded from education.

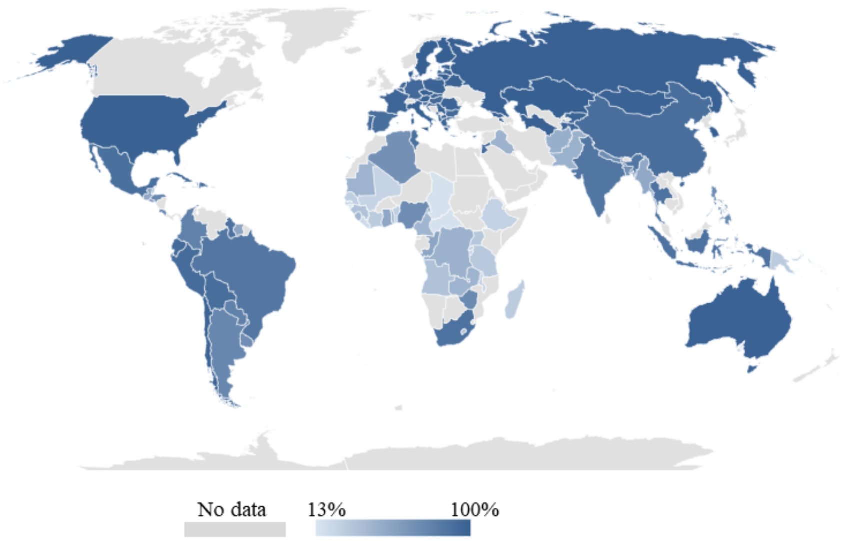


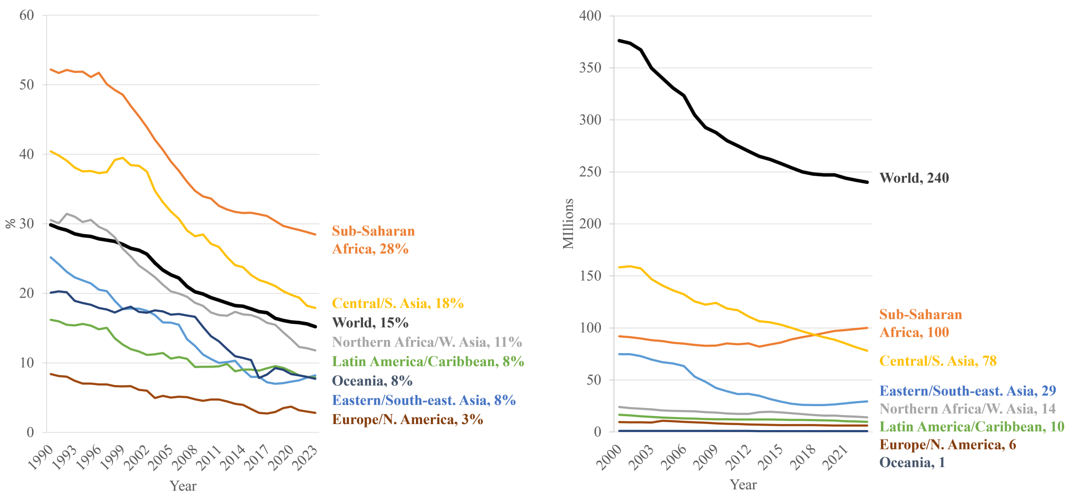
Figure 1.3: Completion rate for lower secondary school students, per country. Data: UNESCO Institute for Statistics (UIS).

The first possible step to promote access to school education is the participation of children in pre-school education (in Italy called "scuola dell'infanzia"). In fact, several studies claim that pre-schooling fosters the development of a learning base that is fundamental for continuation in school. In addition, children and families can realise the importance of education from an early age, supporting further education and decreasing the likelihood of dropping out of school. For this reason, Indicator 4.2.2 monitors the participation rate of children in organized learning one year prior to primary school. In low-income countries, only 43 per cent of children are enrolled in pre-school education the year before primary school, a rate that reaches 75 per cent globally, whereas it stood at 65 per cent in 2002.

Some countries have in fact recorded a sharp increase: in Kyrgyzstan, for example, the participation rate has risen from 42 per cent in 2000 to 84 per cent in 2021; in Laos, it has risen from 10 per cent to 71 per cent over the same period. However, participation remains very low in several sub-Saharan African countries, such as in Senegal where the rate has remained constant in recent years at 16% since 2012.
Another factor to be monitored concerns the age of students in the school system: in several countries, children are older than the students in their grade at the time of enrolment in primary school. This often leads to an increased probability of dropping out of school over the years, as well as to an uneven learning environment. Indicator 4.1.5 monitors the proportion of students who are over-age at enrolment and highlights countries such as Liberia, where one in two (47%) children at age 16 are still in primary school, or Malawi, where only 13% of children at age 15 are in secondary school. It is therefore clear that in these countries there is a need for a school system that is able to include over-age youth in the education system, with a pathway adapted to their needs.
Finally, indicator 4.1.5 addresses the problem of children completely excluded from the school system. Globally, in fact, 240 million children and young people are estimated to be out of school in 2023. As far as primary school is concerned, the Global Monitoring Report on Education together with a new model developed by the UNESCO Institute for Statistics suggests that the out-of-school rate has declined faster than previously thought, while it remains relatively stable when looking at the lower secondary school system. Here again, the difference is substantial with regard to different regions of the world, with an increase in the number of children excluded from the school system in Sub-Saharan Africa, corresponding to an increase in the child population, see Figure 1.4. Indeed, in Sub-Saharan Africa, the school-aged population almost doubled between 2000 and 2022. Consequently, even though the rate of out-of-school children has almost halved during this period, the number of out-of-school children has barely changed.
One of the main reasons behind non-participation in the education system is conflict, as well as poverty and lack of facilities. Moreover, the impact of the Covid 19 pandemic certainly affected pre-school children, who did not have the opportunity to enroll in school, and further studies will be needed to assess how the pandemic will affect them in future years.

Equity

Although there has been global progress towards gender equality in education, significant gaps persist, particularly between individuals in rural versus urban areas and between those who are rich versus poor. Addressing these disparities and ensuring that no one is left behind is a central focus of the Sustainable Development Goals (SDGs), and is specifically highlighted in the fifth target of SDG 4. This emphasis on equity means that policy makers must prioritize assistance to those who are furthest behind.
To effectively monitor progress in closing educational gaps, data must be disaggregated by various characteristics, including income, gender, age, race, ethnicity, migration status, disability, and geographic location, among others. The World Inequality Database on Education has been using household surveys since 2010 to provide insights into the extent of educational disparities in different countries.

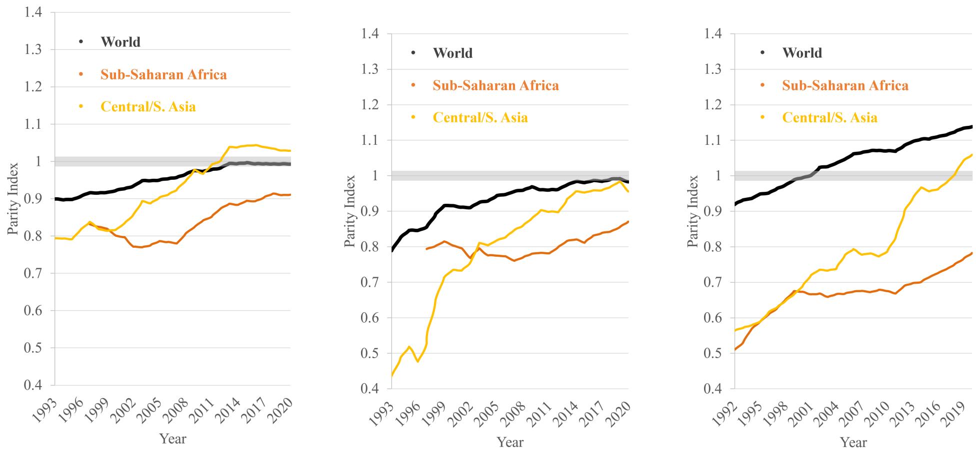


(a) (b)

Figure 1.4: Out-of-school kids, adolescents and youth per region in (a) rate and (b) millions. Data: UNESCO Institute for Statistics (UIS).

With regard to gender, the gender parity index serves as an indicator of the gender gap by measuring gender equality through the comparison of female to male enrollment in education, where a value of 1 represents full gender parity. While there has been significant global progress since 2000, as illustrated in Figure 1.5, persistent gender gaps still exist at the country level. For instance, in 1990 girls in central and southern Asia were significantly lagging behind in education. However, there has been a significant advancement towards achieving gender parity in lower and upper secondary education, accompanied by higher enrollment rates of female students in tertiary education, where India has played a crucial role in achieving these milestones. As a result, the region has now surpassed sub-Saharan Africa in achieving gender parity, with a parity index of 0.96 for upper secondary and 1.06 in tertiary education as of 2020, whereas sub-Saharan Africa still lags behind with a parity index of 0.87 for upper secondary and 0.78 in tertiary education.

When examining education gaps, other disparities, for instance the difference between the rich and poor, should also be taken into consideration. Household surveys are valuable in revealing such gaps, as seen in the case of Pakistan, where 92% of the richest attend primary education compared to only 28% of the poorest, and Nigeria, where 97% of the richest attend lower secondary education compared to only 27% of the poorest. It is also important to consider how these gaps change as children progress through their education: in Mali, for example, gender gaps widen as girls progress to higher levels of education. Similarly, when examining wealth, education gaps usually accumulate over time, particularly for the poorest. Moreover, it is essential to take into account both wealth and individual characteristics, such as gender, as they intersect and contribute to the education gap accumulation over time. For instance, in sub-Saharan Africa, the gaps between the poorest and richest widen as children progress through their education



(a) Lower secondary (b) Upper secondary (c) Tertiary

Figure 1.5: Gender parity index in the period 1993-2020 in different grades for Central/Southern Asia, the world, sub-Saharan Africa.

and depending on the gender, with fewer than 45 of the poorest girls attending tertiary education compared to nearly 80 of the richest boys.

In conclusion, education inequality remains a significant challenge across the world, with significant gaps between different groups within countries. To address this issue, high-quality, disaggregated data is essential to design and implement effective strategies that can help reduce these inequalities and ensure that no one is left behind in the pursuit of achieving SDG 4. It is imperative that policymakers and stakeholders prioritize education equity and work towards closing these gaps to ensure that all individuals have access to quality education and lifelong learning opportunities, regardless of their gender, socioeconomic status, geographic location, or other relevant characteristics. Only by working together can we ensure that education truly becomes a transformative force that empowers individuals and fosters sustainable development for all.

Learning

While it is important to get children enrolled in schools, simply attending school is not enough to ensure that they are learning. Basic reading and math skills are essential for individual empowerment, well-being, and for achieving social, political, and economic outcomes. However, many children in some of the world's poorest countries lack these fundamental skills. Early grade reading assessments in countries such as Nigeria and the Democratic Republic of Congo show that over 80% of grade 3 students cannot even read a single word of connected text. Globally, it is estimated that nearly 40% of primary school-age children do not have basic reading skills, with 43% of children in Central and Southern Asia, and 84% of children in sub-Saharan Africa lacking these skills, compared to only 3% of children in Europe and Northern America.

Moreover, completing primary school is no guarantee that a child will be able to read.

Globally, four out of ten children who completed primary education lack basic reading skills, rising to eight out of ten in sub-Saharan Africa. Gender learning gaps in primary education are relatively narrow, spanning no more than ten percentage points for nearly all countries. However, learning gaps between countries are enormous, with some sub-Saharan African countries reporting only 4% of students attaining minimum proficiency in reading, while others such as Gabon report 76%. Similarly, in mathematics in primary education, rates range from 2% in the Dominican Republic to 39% in Peru. While SDG 4 emphasizes the need to monitor learning, many countries still face obstacles preventing them from adopting effective and cost-efficient solutions to assess and report on learning outcomes. Nonetheless, the available evidence suggests that a vast majority of students in the world's poorest countries are not achieving minimum proficiency in basic skills.

Quality

The quality of education is a multifaceted concept that extends beyond learning outcomes. In addition to having a strong academic foundation, students need to have access to well-trained teachers, a safe and supportive learning environment, and adequate infrastructure, among other things.

Teachers are a critical component of quality education, and the quality of a teacher's training and preparation is essential for ensuring that they are able to effectively deliver instruction to their students. However, in many countries, the number of teachers has not kept pace with the increase in the number of students, leading to overcrowded classrooms and a decline in the quality of education. There are two main ways that countries measure the quality of their teachers. The first is by assessing whether teachers are qualified, meaning that they have the necessary academic credentials according to national standards. The second is by assessing whether teachers are trained, which looks at whether they have received appropriate pedagogical and professional knowledge. Despite the importance of teacher training for quality education, the definition of what constitutes trained teachers can vary greatly between countries, and there is often a lack of data to compare and track progress. While the majority of teachers around the world are qualified, trained, or both, the percentage of trained teachers in sub-Saharan Africa has decreased over time, from 84% in 2000 to 69% in 2019.

Ensuring a quality learning environment is another fundamental aspect for providing effective education. However, achieving this goal is a challenge in many countries, particularly in sub-Saharan Africa. Inadequate infrastructure, including lack of water, sanitation, and electricity, can seriously hinder the ability of students to learn. For example, lack of appropriate sanitation facilities can prevent girls from attending school, and lack of electricity can limit access to technology and modern teaching methods. Moreover, ensuring a safe and non-violent learning environment is also essential. However, measuring progress on these issues is difficult due to the lack of internationally comparable data on teacher indicators and the fact that the relevant global indicator encompasses a set of several dimensions. Addressing these challenges and improving infrastructure and learning environments is essential for providing children with the quality education they need to succeed in life.

Finance

The achievement of the ambitious SDG education targets is contingent upon additional resources, particularly in countries that are the furthest behind. The GEM report suggests that low- and lower-middle-income countries need to spend 504 US$ billion or 6.3% of their gross domestic product (GDP) annually in order to achieve the 2030 target for universal basic education, comprising pre-primary, primary, and secondary education, UNESCO (2020). However, having these resources may not be enough; countries spending the same amount in terms of income per capita can have disparate education outcomes depending on whether they are spending effectively, efficiently, and equitably. Out of 78 low- and middle-income countries with available information, only 17 maintained a strong equity focus through financing policies, primarily upper-middle-income and Latin American countries. The main sources of education spending are governments, households, and donors. While governments globally have adhered closely to the finance benchmarks set by the Education 2030 Framework for Action, poorer countries with low capacity to generate domestic resources tend to spend less on education as a share of GDP but more as a share of total government spending. Aid thus become important source of education financing in low-income countries, which must be analysed jointly with domestic public and private funding. In addition to increasing the funds available for education, equitable distribution needs to be strengthened. In order to achieve the global education goal, SDG 4, it is crucial that countries and donors take responsibility and fulfil their commitments.

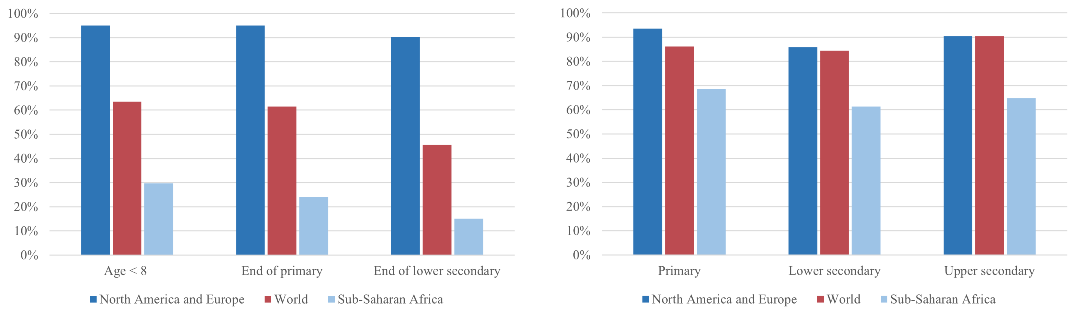
1.3 The Role of Mathematics

Mathematics education is a crucial component of sustainable development, as it promotes scientific and technological progress, fosters economic growth, enhances critical thinking, problem-solving skills, and facilitates social mobility and equity. However, there are significant disparities in math achievement across various countries and regions, as evidenced by the Programme for International Student Assessment (PISA) 2018 report. The report indicates that China, Singapore, and Hong Kong are among the top performers in math, while the Dominican Republic, Morocco, and Honduras are among the lowest performers. Such differences in math achievement are influenced by factors such as access to quality education, teacher training and support, and socio-economic status.

To address these underlying factors and improve math education in low-performing countries and regions, there is a need to ensure access to quality education and training for teachers to deliver effective math instruction. The Global Indicator 4.1.1 has been established to monitor the percentage of students achieving a minimum level of proficiency in reading and mathematics at the end of primary education and lower secondary education. However, it is essential to keep in mind that good quality education extends beyond mere learning outcomes: many children worldwide lack access to good school infrastructure, or a safe and nonviolent learning environment, which hinders their ability to succeed academically.

According to data, there is a significant disparity in mathematics achievement between students in North America and Europe, and sub-Saharan Africa. While 95 percent of

students under the age of 8 in North America and Europe attain the expected basic level of mathematics, only 29.6 percent of sub-Saharan African students achieve the same. This trend persists as students progress through primary and lower secondary education, with only 24 percent and 15 percent of African students, respectively, reaching the expected minimum level of proficiency in mathematics, see Figure 1.6a. One contributing factor to this discrepancy is the qualification and training of teachers. In North America and Europe, 95 percent of teachers are qualified and trained, while in sub-Saharan Africa, only 69 percent of primary teachers and 61 percent of secondary teachers have the same level of qualification and training, see Figure 1.6b. It is evident that promoting quality education, particularly in mathematics, is a complex and multifaceted endeavor that requires addressing various factors and intervention, such as teacher training, curriculum development, and resource allocation, among others.
There are successful programs and interventions that have been implemented to improve math education in low-performing countries. The Mindset program in South Africa, for instance, provides teacher training and support to deliver effective math instruction and has been shown to improve math scores and decrease the achievement gap between advantaged and disadvantaged students. These programs offer hope for effective solutions to improve math education in low-performing countries and regions. However, addressing the underlying issues of teacher training and access to quality education remains a complex and ongoing challenge that requires continued attention and resources. By taking a comprehensive approach and implementing successful programs and interventions, it is possible work towards providing all students with the necessary tools to succeed in math and beyond.



(a) Minimum mathematics skills (b) Percentage of qualified teachers

Figure 1.6: Comparison of data from the UNESCO Institute for Statistics (UIS) on the achievement of minimum mathematics skills (a) and the percentage of qualified teachers (b) for North America and Europe, the World, and sub-Saharan Africa.

In addition, european countries face specific challenges related to increasing numbers of migrants and refugees in the classroom context. Migrant and refugee students may face language barriers, cultural differences in teaching and learning styles, and socio-economic challenges, all of which can affect their ability to access quality math education. According to a report by the European Commission, in 2019, more than 1.3 million first-time asylum seekers applied for international protection in the EU, which poses significant challenges for the education sector. For instance, in Germany the number of refugee students has increased significantly in recent years, and many of these students have experienced disruptions to their education due to war and conflict in their home countries, and may also face language barriers and cultural differences in the German education system. A case study conducted in Germany by Grüttner et al. (2018) found that refugee students in the city faced significant challenges in accessing quality math education from a capabilities and engagements perspective. The study found that these students often had lower levels of prior math knowledge and struggled with the language used in math textbooks and classroom instruction. In addition, some teachers had limited experience working with non-native speakers of German, which made it difficult to provide appropriate support for these students. The study also identified a lack of continuity in math education for refugee students, with some students experiencing long gaps in their math education due to displacement and resettlement. As a result, these students often struggled to catch up with their peers and faced difficulties in advancing to higher-level math courses. To address these challenges, the study recommended providing targeted support for refugee students in math, including additional language support and more individualized instruction. It also suggested that teachers receive training in working with non-native speakers of German and in providing culturally responsive instruction. This includes using real-world examples that are relevant to the students' experiences and cultural backgrounds, and providing opportunities for peer-to-peer learning and collaboration. Overall, the study highlights the importance of addressing the unique needs of migrant and refugee students in math education in order to promote their academic success and social inclusion.

While contextual factors such as politics, resources, and culture may give the impression that the challenge of education varies from one country or group to another, the underlying challenge remains the same, regardless of the context. Educational systems need to overcome cultural barriers, improve learning outcomes, and enhance the overall learning experience for all students. This thesis specifically focuses on addressing the challenges of math education in sub-Saharan Africa by improving students' engagement and understanding of mathematics. While territorial, infrastructure and financial limitations often accompany education in this region, this thesis does not aim to address these aspects but rather explores innovative approaches that can enhance the overall learning experience and promote positive attitudes towards mathematics. To achieve this goal, innovative approaches such as games, technology, and project-based learning are taken into consideratio in order to enhance students' engagement with mathematics. Incorporating interactive and gamified elements into the teaching of mathematics can help students develop a more positive attitude towards the subject and motivate them to learn. Additionally, ethnomathematics, which explores the cultural aspects of mathematics, can provide a new perspective and appreciation for the subject, both in European and intercultural contexts. The following sections will delve into these approaches, discussing their benefits and how they can be effectively implemented in the context of the 2030 Sustainable Development Agenda.

1.3.1 Games, Technology and Project Based Learning

Many students around the world perceive mathematics as a complicated and confusing subject due to its use of formulas and calculations, and some find it boring and irrelevant to their real-life situations. Conventional learning tools such as textbooks, revision books, and courseware are not very effective in helping students master the subject. In general, many students express frustration with mathematics education and traditional learning tools, highlighting several common problems. Firstly, students often lack motivation to study mathematics and fail to see the benefits of learning it. Secondly, conventional learning tools can be unengaging and fail to provide opportunities for interaction with the material. Thirdly, these tools do not encourage self-learning or provide enough stimulation for students to explore their own questions. Fourthly, the material covered may not be relevant to students' everyday lives, and finally, the lack of continuity in learning further hinders student progress. This is even more true in low-income countries and specifically in sub-Saharan Africa, where students face additional challenges such as limited resources and cultural barriers. Moreover, many students in these regions often fail to recognize the relevance of mathematics in their daily lives, and they may not have role models or career opportunities in STEM to inspire and motivate them. This can lead to an overall lack of motivation and disengagement from scientific subjects. As a result, innovative approaches that promote a more positive attitude towards mathematics and enhance students' engagement are crucial to improving learning outcomes in these contexts. Research has shown that people learn best when they have a strong and immediate motivation to acquire new knowledge and when they are having fun, Gee (2003). Examples of such innovative approaches that promote a more positive attitude towards mathematics and enhance students' engagement include game-based learning, technology, and project-based learning. In the following section, we will delve into the literature review of these approaches and discuss their benefits, in order to have a clearer idea on how they can be effectively implemented in the context of sub-Saharan Africa.

Game-based learning

Game-based learning is a promising approach to teaching and learning that has gained significant attention in recent years. In the field of mathematics education, game-based learning involves the use of games and simulations as tools for education, and has been explored as a mean to address various challenges associated with the teaching and learning of mathematics, including low student motivation, poor conceptual understanding, and difficulties with problem-solving. In terms of methodology, game-based learning approaches in mathematics education typically involve the use of digital or physical games, simulations, or gamification techniques to enhance learning experiences. These games may be designed to support the development of specific mathematical concepts, such as arithmetic or algebraic thinking, or to address more general goals, such as problem-solving or critical thinking skills. Game-based learning environments may be used as standalone teaching tools, as supplements to traditional instruction, or as part of a blended learning approach that combines face-to-face and online instruction. Additionally, game-based learning may involve collaborative or competitive play, individual or group work, and various forms of feedback and assessment to support learning and monitor progress. The selection and design of game-based learning environments may depend on a range of factors, including the learning objectives, the target audience, the availability of resources, and the technological infrastructure.

This section will provide an overview of the literature on game-based learning in mathematics education, focusing on the various perspectives that have been adopted in research studies and in literature.

The first aspect that has been explored in research on game-based learning in mathematics education is its effectiveness on student's performance. A study titled "Digital Game-Based Learning for Remedial Mathematics Students: A New Teaching and Learning Approach in Malaysia", conducted by Syed Hussain et al. (2017), investigates the effectiveness of game-based learning in supporting remedial mathematics students in Malaysia. The study aims to determine whether a specific use of digital games can impact the performance of remedial students. The analysis demonstrated that game-based learning offers an engaging and interactive learning experience, which results in a more fruitful dedication to the subject. The study found that students who participated in the game-based learning approach demonstrated significant improvement in their mathematical abilities compared to those who received traditional instruction. These findings suggest that game-based learning can be an effective teaching and learning approach, especially for students in situations where traditional teaching methods may not be effective. Similarly, Brezovszky (2018) investigated the effectiveness of a mathematics game-based learning environment in promoting adaptive number knowledge and arithmetic skills among primary school students. The study included 1168 participants from grades four to six, who were randomly assigned to either the experimental or control group. The experimental group received regular mathematics teaching enriched with gameplay using the Number Navigation Game (NNG), while the control group followed their regular math curriculum. The study utilized an experimental design with pre- and post-tests to assess the students' adaptive number knowledge, arithmetic fluency, and pre-algebra knowledge. The findings revealed that the experimental group performed better than the control group on adaptive number knowledge and math fluency. The results also indicated varying effects of the training on different grade levels, with grade five students showing more significant improvement in their adaptive number knowledge.

Another aspect that has been explored in research on game-based learning in mathematics education is its influence on student motivation and involvement. However, evaluating student motivation and involvement can be challenging since it is often influenced by factors such as the liking of the specific game or the novelty of the technology used. Nonetheless, research has consistently shown the potential of game-based learning in promoting positive attitudes towards mathematics and improving learning outcomes. In general, game-based learning provides students with a fun and interactive environment that can help them better understand and enjoy mathematical concepts. Ahmad et al. (2015) conducted a study to investigate the effectiveness of using a role-playing game as a supplementary tool for teaching mathematics, with a particular focus on decimals, which is a challenging topic for many school children. After some practices, a questionnaire has been conducted in order to assess learnability, satisfaction, screen design, and performance effectiveness. The participants gave an average mean score of 4.3 out of 5, indicating a satisfactory level of satisfaction with the prototype. The study found that the use of the game-based learning approach increased students' engagement and interest in the topic of decimals, suggesting that game-based learning can be an effective means of promoting student motivation and involvement in mathematics education.

Other researches on game-based learning has focused on the influence of the tool in the emotional and affective dimensions of learning mathematics, while some other has focused on the use of different representations and visualizations in game-based learning environments, which can provide students with alternative ways of understanding mathematical concepts. Overall, these different perspectives on game-based learning in mathematics education highlight the potential of this approach to promote students' motivation, outcomes, and conceptual understanding of mathematical concepts. By providing students with interactive and stimulating learning environments that incorporate a variety of representations and visualizations, game-based learning has the potential to improve the quality of mathematics education and enhance students' mathematical competencies.

Technology

In recent years, educational technology has become an integral part of education, where the term educational technology refers to various technology-based programs or applications that support the delivery of learning materials and aid the learning process. Examples include computer-assisted instruction (CAI), integrated learning systems (ILS), and technology-based curricula. As the world becomes increasingly digitized, the use of technology in education is becoming more prevalent and essential. This is especially important in low-income countries, where access to technology and digital resources is often limited to the school context, but is fundamental to prepare students for future careers and work. Fortunately, technological advancements have made it more cost-effective to implement these tools in the classroom, allowing educators to incorporate technology into their teaching practices more easily. Therefore, the question is no longer whether teachers should use educational technology, but rather what impact its implementation has on students' achievements, attitudes, and the correlation between the two.

A systematic review and meta-analysis by Cheung and Slavin (2013) investigated the impact of educational technology applications on mathematics achievement in K-12 classrooms. The study examined 74 qualified studies, most of them from United Stated, with a total sample size of 56886 K-12 students, including 45 elementary school studies and 29 secondary school studies. Overall, the findings revealed a positive yet modest effect (ES = +0.15) of educational technology applications on mathematics achievement compared to traditional teaching methods. However, the effectiveness varied depending on the type of educational technology used, with larger effect sizes observed for studies that involved longer intervention periods and individualized instruction. The authors highlighted that educational apps could be a valuable tool to enhance mathematical understanding, especially for students who struggle with conventional teaching approaches or come from disadvantaged backgrounds. Nonetheless, they cautioned that the quality of educational apps varies considerably, and further research is needed to identify the most effective types of apps and their integration into classroom instruction.

With regards to students' attitude towards mathematics, the study by Attard and Holmes (2013) explores how technology can be used to enhance student engagement with the subject. The authors conducted a qualitative study involving interviews with high school students and teachers who used technology in their mathematics classes in Australia. The study findings showed that technology-based practices were instrumental in promoting student engagement by facilitating positive pedagogical relationships (i.e. interpersonal educational relationships between teachers and students) and creating engaging pedagogical repertoires (i.e. the teaching practices that a teacher chooses on a daily basis.). Moreover, the findings suggest that if teachers gain a deeper understanding of the unique pedagogical benefits that various technological tools offer, they may be able to further enhance student engagement with mathematics. Ultimately, this increased engagement may lead to a greater desire among students to continue learning and expanding their mathematical knowledge beyond the compulsory school years.

In addition to these outcomes, technology has the potential to personalize learning, allowing students to progress at their own pace and receive immediate feedback, thereby enhancing their mathematical understanding. Adaptive learning software can be used to adjust the difficulty level and content of learning activities based on the student's performance and learning needs. This allows students to receive individualized support that can improve their understanding and retention of mathematical concepts. This is particularly important in sub-Saharan Africa, where the teacher-pupil ratio is often high, making it difficult to provide individualized attention to each student. An example of this application is the open source online assessment package called STACK (System for Teaching and Assessment using a Computer algebra Kernel) developed in collaboration with IDEMS for Maseno University, Kenya. STACK has the ability to generate randomized versions of questions, allowing students to enter a mathematical expression as answers, and predict common mistakes, providing relevant feedback to the students. The main goal is to make it a more personalized and effective way of assessing mathematical understanding, and is now implemented by universities and schools both in and outside the African context.

Project-based learning

Project-based learning is a pedagogical approach that can provide students with opportunities to apply mathematical concepts in real-world situations or to create a product, engaging them in the learning process. Through project-based learning, students can develop critical thinking and problem-solving skills, as well as gain hands-on experience in applying mathematical concepts in a meaningful way. The philosophy of PjBL considers that learning is more engaged when triggered by a student's "I need to know" than by a teacher's "because you should know" (Lenz et al., 2015). Moreover, projects often require students to collaborate, communicate, and think critically, which can help develop problem-solving, interpersonal and teamworking skills.

Main research on project-based learning primarily centers around its effects on student achievement and cognitive skills. According to the findings presented in the meta-analysis by Chen and Yang (2019), project-based learning had a medium to large positive effect on academic achievement, with an overall mean weighted effect size of 0.71. The authors analyzed 60 studies with a total of 10628 participants from different countries and

grade levels. The studies included various types of projects, such as science experiments, engineering designs, and multimedia presentations. The authors also looked at different moderators, such as age, gender, subject area, and duration of the intervention, to see if they influenced the effectiveness of project-based learning. The study revealed that students who participated in project-based learning exhibited higher achievement scores and reported greater levels of motivation in comparison to those who received traditional instruction. The authors suggest that this effect may be due to the active and inquiry-based nature of project-based learning, which can lead to deeper understanding and retention of concepts.
The study by Anazifa and Djukri (2017) investigated instead the effectiveness of project-based learning and problem-based learning in improving students' thinking skills. The research question focused on whether these two methods are effective in enhancing students' critical thinking, problem-solving, and decision-making skills. The study involved 86 high school students in Indonesia who were randomly assigned to either a project-based or problem-based group. The students worked on tasks related to entrepreneurship, such as developing a business plan or creating a product prototype. The authors used a pre- and -post test design to measure changes in students' critical thinking, problem-solving, and decision-making skills. The results of the study showed that both project-based learning and problem-based learning were effective in improving students' thinking skills, with project-based learning showing slightly better results. The findings suggest that these approaches can be valuable tools for educators seeking to develop students' higher-order thinking skills. However, it is important to note that the effectiveness of these approaches may depend on a range of factors, including the nature of the learning environment, the quality of teacher support and guidance, and the characteristics and needs of individual students.

Despite the numerous positive impacts that games, technology, and project-based learning have been shown to have on students' achievements, attitudes, and engagement, criticisms have also been raised regarding their implementation. While technology has been demonstrated to be useful for at-risk students, it may not be suitable for all students and may hinder the development of social skills and face-to-face interaction if not used correctly. Moreover, in low-income areas where access to technology may be limited, relying solely on these tools could exacerbate the achievement gap. Critics argue that an excessive emphasis on technology could prevent students from developing critical non-digital skills, such as communication, collaboration, and creativity. Therefore, striking a balance between traditional teaching methods and integrating technology and games into the curriculum is essential to enhance student learning and engagement without sacrificing essential skills development. Adequate training for teachers and meticulous planning are necessary to ensure the effective, efficient, and equitable implementation of technology and project-based learning.

In the next chapter of this thesis, a novel approach to teaching mathematics through the use of games, technology, and project-based learning is presented. The proposed initiative is a Math Camp held in Kitale, Kenya, which aims to promote mathematics and introduce students to the use of technology and digitalization. The Math Camp offered an ideal setting to implement these approaches in teaching mathematics, providing a hands-on and interactive learning experience that fostered critical thinking and problem-solving skills. The use of games and physical activities was fundamental in enhancing the learning process and creating a community environment that facilitated social interaction and teamwork. By introducing students to new technologies and digital tools, the Math Camp sought to prepare them for the challenges and opportunities of the digital age. The chapter will present an evaluation and feedback of the camp, including an example of a mathematical activity implemented during the camp.

1.3.2 Ethnomathematics

In the late 1980s, the concept of intercultural approaches in mathematics education emerged, defining mathematics not only as an academic subject but also as a set of techniques and arts developed by diverse cultures to explain, understand, and address their environment. The term "ethnomathematics" was coined by the Brazilian scholar Ubiratàn D'Ambrosio in 1977, from the etymology of three Greek roots: ethno (*éthnos*, people), matema (*máthema*, knowledge, learning), and tics (*téchne*, art in the sense of "know-how"). Ethnomathematics studies the reasons why members of specific cultures (*éthnos*) have developed over time techniques and ideas of measurement, calculation, deduction, comparison, and classification (*téchne*) that allow them to model natural and social environments and explain and understand phenomena (*máthema*). According to D'Ambrosio, ethnomathematics is a program that combines mathematical ideas and procedures from different cultural groups, not only as ethnic groups but also as groups of belonging, such as workers in a particular industry, professional classes, or children of the same age group. The program represents a methodology for researching and analyzing the processes that transmit, spread, and institutionalize mathematical knowledge, defined as ideas, processes, and methods. The goals of ethnomathematics researchers are to improve mathematics learning through a rediscovery of each individual's cultural baggage and to increase cultural integration, understood both in a multiethnic sense and in a transversal sense with respect to different groups of belonging. The idea is to offer an innovative theoretical basis regarding the development of mathematical knowledge and the understanding of human behavior through the study of mathematical ideas and procedures practiced by humanity. The program of ethnomathematics aims to analyze the social and cultural origins of mathematical knowledge by exploring six dimensions, including cognitive, cultural, educational, epistemological, historical, and political dimensions, as proposed by Rosa et al. (2017). This approach aims to emphasize cognitive processes, learning abilities, and attitudes that may influence the learning process in classrooms, while also appreciating the historical and political relationships among different groups that have contributed to the development of modern mathematics.

In 1985, the International Study Group on Ethnomathematics (ISGEm) was founded during the annual meeting of the National Council of Teachers of Mathematics in the United States. This program of research in the history and philosophy of mathematics, with a focus on pedagogical implications, aimed to increase the understanding of cultural diversity in mathematical practices and to apply this knowledge to education, sustainable development, and social justice. The ISGEm has organized seven International Conferences on Ethnomathematics (ICEM) since its inception, involving representatives from over 20 nations and international academic institutions. Furthermore, since 1988, the ISGEm has promoted discussion groups in all International Congresses on Mathematical Education (ICME), organized by the International Commission on Mathematical Education, an organization dedicated to the research and development of mathematics education at all levels since 1908. The International Study Group on Ethnomathematics identifies four general areas of interest in ethnomathematics (Rosa et al., 2016):

1. Field research and data acquisition from different cultural groups: this area focuses on collecting experimental data in the field. Researchers, who can be either outsiders or members of the local culture, observe and study the mathematical activities practiced by the research group, seeking to identify cases of mathematical thinking. The results can be numerical or geometric, linguistic, describing terms used in mathematical applications, or can describe a cultural structure or an engineering process for solving problems of daily life. The goal is to understand the mathematical practices of different cultures and to promote cultural diversity in mathematics education.

2. Applications of ethnomathematics in the classroom: this area focuses on applying ethnomathematics in the classroom. The goal is to introduce students to the mathematical practices of different cultures and to use these practices as a way of promoting multiculturalism and social justice.

3. Mathematical and didactic work in intercultural situations: this area focuses on developing mathematical teaching and learning practices that are appropriate for intercultural situations. The goal is to develop teaching methods that are sensitive to the cultural backgrounds of the learners and that promote effective learning in diverse cultural contexts.

4. Theoretical, sociological, and political studies on ethnomathematics: this area focuses on examining the theoretical foundations of ethnomathematics, including reflections on the future prospects of the program and the areas where further research is needed. The goal is to develop a deeper understanding of the relationship between mathematics and culture, and to explore the social and political implications of this relationship.

The first exceptional example of early ethnomathematical research is provided by Ascher (1991) "Ethnomathematics: A Multicultural View of Mathematical Ideas". In the book, Ascher examines the mathematical ideas of people from various traditional or small-scale cultures, such as the Tshokwe, Bushoong, and Kpelle of Africa, the Inuit, Navajo, and Iroquois of North America, the Malekula, Warlpiri, and Maori, among others. She provides examples of how these cultures use mathematics in their everyday lives, as well as how their mathematical knowledge and practices can be integrated into the mathematics curriculum. The book also raises important questions about the relationship between mathematics and culture, and challenges us to broaden our understanding of what constitutes “legitimate” mathematical knowledge.

In the following paragraph, an example of ethnomatematics application by Paulus Gerdes is presented, in order to illustrate how an ethnomatematical activity could be implemented.

Literature case study: the use of Sona

Paulus Gerdes was a mathematician at the Eduardo Mondlane University and the Universidade Pedagogica in Mozambique who studied the mathematics embedded in African cultures. One of his earliest studies focused on the use of Sona, traditional drawings on sand, in mathematics education. Sona is a form of geometric art that is traditionally created by the Yao people of southern Africa, see Figure 1.7. Gerdes was interested in exploring how the creation and interpretation of Sona can be used to teach geometry and spatial reasoning.

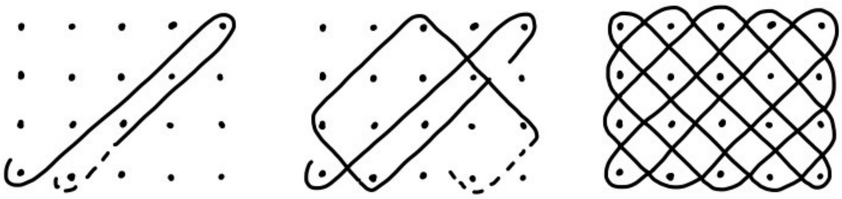


Figure 1.7: Example of a Sona drawing.

Gerdes (1988) conducted an early study on the possibility of incorporating traditional sand drawings by the Tchokwe people in northeast Angola into math classes. These sand drawings, known as Sona, are a crucial element of the Tchokwe’s cultural heritage, used to transmit knowledge and wisdom from one generation to the next. To aid in the memorization of the pictograms, the “akwa kuta sona”, or drawing experts, developed a mnemonic device involving an orthogonal grid of equidistant points traced on the ground with their fingers. This method effectively reduced the memorization of an entire drawing to two numbers and a geometric algorithm, using an early form of a coordinate system. Gerdes made significant contributions to the reconstruction of mathematical concepts involved in Sona, ranging from arithmetic relationships, progressions, symmetry, similarity, Euler graphs, to the determination of the greatest common divisor of two natural numbers. One notable property he demonstrated is that the number of lines needed to complete each Sona corresponds exactly to the great common divisor (gcd) of the two positive numbers representing the dimensions of the Sona, following a short list of drawing rules. Gerdes affirms that incorporating Sona into math classes can help achieve important social goals, such as reviving, strengthening, and valuing this practice among the Tchokwe people, which was threatened with extinction during colonial occupation. Furthermore, it can contribute to a national curriculum that values this practice and fosters a sense of national culture, and it can help international curricula appreciate the unique mathematical ideas, thoughts, and methods developed by different peoples.

A more recent study by Maffei and Favilli (2004) builds upon Gerdes' pioneering research on Sona and provides further evidence for the effectiveness of using culturally relevant materials and techniques in mathematics education. In their study, they worked with a group of 42 Italian students in the first year of secondary school who had no previous exposure to Sona or other traditional African art forms. The study involved a pre-test and a post-test to assess the students' geometric thinking skills before and after the Sona intervention. The pre-test consisted of 10 multiple-choice questions, while the post-test consisted of 12 open-ended questions that required the students to explain their reasoning and problem-solving strategies. The results of the study showed a significant improvement in the students' geometric thinking skills after the Sona intervention. The students were able to use their Sona drawings to develop a deeper understanding of geometric concepts such as symmetry and reflection, and were able to apply these concepts to solve more complex geometric problems. Maffei and Favilli also conducted a survey to assess the students' attitudes towards the use of Sona in mathematics education. The results showed that the students were highly engaged and motivated by the use of Sona, and that they appreciated the cultural relevance of the activity. Overall, the study by Maffei and Favilli provides compelling evidence for the effectiveness of using culturally relevant materials and techniques in mathematics education. By incorporating traditional African art forms such as Sona into the classroom, the cultural awareness and appreciation of the classroom increased, as well as the development of students' mathematical thinking and problem-solving skills.

Ethnomatematics and education

Ethnomathematics has been a subject of discussion in the field of mathematics education, and while it has been embraced by many educators and scholars, it is not without its detractors. Some critics argue that ethnomathematics essentializes culture and reinforces stereotypes, or that it is presented as a separate field or domain, rather than an integral part of mathematics education. Additionally, some scholars argue that ethnomathematics is often treated as a "bolt-on" to traditional mathematics curricula, rather than being integrated into the broader context of mathematical inquiry (Rowlands and Carson, 2002). This separation can lead to the marginalization of ethnomathematics and reinforce the idea that it is a peripheral or exotic topic rather than a valuable aspect of mathematical knowledge. Moreover, they argue that focusing too much on cultural particularities could detract from the universal aspects of mathematics, promote cultural relativism and lead to a rejection of formal academic mathematics. According to this view, mathematics becomes a matter of subjective cultural interpretation, rather than a universal and objective field of inquiry.

On the contrary, proponents of ethnomathematics argue that the focus on cultural particularities can actually enhance students' understanding of the universal aspects of mathematics by highlighting the diversity and complexity of mathematical knowledge systems

across cultures. They argue that students can develop a more nuanced and sophisticated understanding of mathematics by exploring the ways in which different cultures have developed mathematical concepts and practices in response to their social and environmental contexts. Additionally, the shared approach of most current ethnomathematics scholars is to establish a “bridge” between local knowledge and school knowledge, not to deny or replace formal mathematics. This bridging of local mathematical knowledge and formal school mathematics is seen as a way to valorize students’ cultures and, at the same time, allow students to better understand formal mathematics starting from their still-unformalized knowledge. The approach is to integrate original mathematical concepts and practices from students’ cultures with conventional and formal academic mathematics. The students’ mathematical experiences are used to understand how mathematical ideas are formulated and applied. This general mathematical knowledge is then used to introduce conventional mathematics in a way that helps students better understand it, appreciate its power, beauty, and utility, and make its relationship with familiar concepts and practices explicit. In other words, a program of this type allows students to become aware of how people mathematize and use this awareness to learn a wider mathematics.
The third chapter of this thesis work focuses on the implementation of ethnomathematics activities in the South Sudanese context. In this case, the use of ethnomathematics is justified by the students’ need to see the relevance of mathematics in their lives, especially in a country where the education system and academic achievements are facing significant challenges. The benefits and potentiality of ethnomathematics in the South Sudanese context will be analysed and supported by some feedback from the students.

Chapter 2

Mathematics Education in Kenya

Kenya's education system is focused on providing universal access to quality education and overseen by the Ministry of Education, Science and Technology. The country has made significant progress in expanding access to primary education, with a net enrollment rate of 83.2% in 2018. Mathematics is a core subject at all levels of education in Kenya, and the aim of mathematics education is to develop critical thinking skills and problem-solving abilities in students, as well as to provide a foundation for further studies in STEM fields. The Kenya Institute of Curriculum Development (KICD) oversees the development and implementation of the national curriculum for mathematics.

However, there are disparities in mathematics proficiency rates by gender and location. According to the UIS, in 2020 the proportion of children/young people at the age of primary education prepared for the future in mathematics for male in Kenya were 58%, while for female students it was 47%. Similarly, for lower secondary school students, the proficiency rates in mathematics for male students were 54%, while for female students it was 41%. These data indicate a gender gap in mathematics proficiency in Kenya, with male students outperforming female students. There are also significant disparities in mathematics proficiency by location in Kenya, see Figure 2.1. According to a report by Uwezo, a non-profit organization that conducts research on education in East Africa, in 2016 only 24% of children in rural areas of Kenya were proficient in mathematics, compared to 63% of children in urban areas (Uwezo, 2016). The proficiency rates vary greatly between different counties, with Nairobi County having the highest proficiency rate at 75%, while Turkana County's proficiency rate is only 13%. These disparities highlight the need for interventions that specifically target the needs of students in rural areas, such as increased access to quality mathematics instruction, teacher training, and learning resources.

Regarding the transition to higher education, the gross enrollment rate (GER) in tertiary education has been steadily increasing, with a rate of 7.2% in 2018. However, there are significant gender disparities in tertiary education, with a GER of 9.1% for males and 5.2% for females in the same year. These disparities are also reflected in the fields of study chosen by male and female students. Females tend to be underrepresented in STEM fields, including mathematics. Initiatives have been put in place to encourage more female students to pursue studies in STEM fields, but socioeconomic status also affects access to

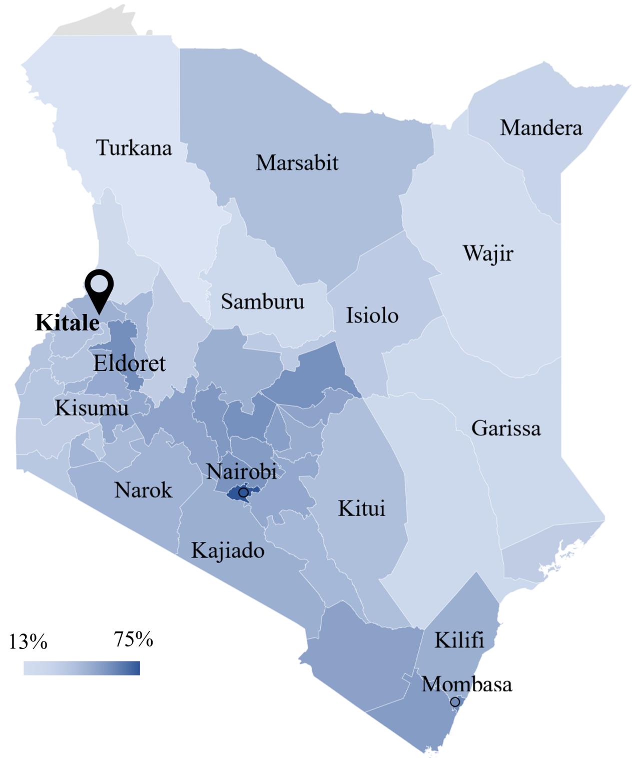


Figure 2.1: Math proficiency rate in per county

tertiary education, with students from wealthier families more likely to attend university. Additionally, there are concerns about the quality of education in some tertiary institutions, which may impact the outcomes and achievements of students in mathematics and other subjects.

The level of computer literacy in Kenya has been increasing over the past decade, according to the Kenya National Bureau of Statistics (KNBS). In 2019, 41.4% of individuals aged 15 years and above reported being computer literate, up from 29.7% in 2015 (Kenya National Bureau of Statistics, 2019). However, there are still significant disparities in computer literacy rates by age and location. Computer literacy rates are highest among younger age groups, with 79.2% of individuals aged 15-24 years reporting being computer literate in 2019, compared to 8.3% of individuals aged 65 years and above. There are also significant disparities in computer literacy by location in Kenya, with only 22.6% of households in rural areas having access to the internet in 2019, compared to 51.5% of households in urban areas, according to a report by the World Bank. The Kenyan government has been actively promoting the adoption of digital technologies in various sectors, including education, and launched the Digital Learning Program in response to the COVID-19 pandemic to provide digital devices and e-learning resources to students in primary and secondary schools. However, there are still challenges in terms of digital access and infrastructure, particularly in rural areas.

2.1 Mathematics initiatives in Kenya

Kenya has implemented several initiatives aimed at improving mathematics education in the country. Here are a few examples:

- The Mathematics and Science for Sub-Saharan Africa (MS4SSA) initiative: This initiative, launched in 2012, aims to improve the quality of mathematics and science education in sub-Saharan Africa, including Kenya. The initiative includes training for teachers, development of teaching materials, and support for the implementation of the national curriculum.

- The Mathematics Centre of Excellence (MaCoE): This initiative, established in 2013, is a partnership between the University of Nairobi and the African Institute for Mathematical Sciences (AIMS). Its aim is to develop the mathematical skills of students and teachers, particularly in disadvantaged areas of Kenya. MaCoE provides training for teachers, as well as afterschool and holiday programs for students.

- The Kenya Education Network (KENET): KENET is a non-profit organization that aims to improve the quality of education in Kenya through the use of technology. KENET has developed several initiatives to support mathematics education, including the use of online resources and the provision of training for teachers in the use of technology to support teaching and learning.

- The Presidential Digital Talent Programme (PDTP): Launched in 2014, the PDTP aims to develop the digital skills of young graduates in Kenya, including in the areas of mathematics and other STEM fields. The program provides training and internships for selected graduates, as well as opportunities for them to work on projects aimed at addressing societal challenges.

These initiatives are just a few examples of the efforts being made to improve mathematics education in Kenya. By doing so, these initiatives aim to develop a highly skilled workforce with strong critical thinking and problem-solving skills, to address skills gaps, and to promote equity by providing opportunities for students from disadvantaged backgrounds to improve their mathematics skills. Overall, the aim is to build a more prosperous and equitable society through improved mathematics education.

To address the challenge of improving the quality of mathematics education in Kenya, I have undertaken a project to organize a math initiative in Kitale, a town located in Trans-Nzoia County, Rift Valley region. With a population of approximately 105000 people and an average age of around 20 years old, Kitale is an ideal location for the initiative. In 2019, according to data from the Kenya National Bureau of Statistics (KNBS), the net enrollment rate in primary education in Trans-Nzoia County was 84.3%, and the proficiency rate in mathematics reached 34.4%, indicating that students in the county scored above 50% in the KCSE exams. However, computer literacy rates in the county

were low, with only 11.8% of the population being computer literate, and the internet penetration rate was just 5.5%, well below the national average of 43.9%. Despite these challenges, the math initiative aimed to enhance mathematics education and computer literacy in Kitale and surrounding areas, and ultimately contribute to improving education outcomes in Kenya.
In the following section, I will go more in detail about this initiative, including its objectives, activities, and expected outcomes. The initiative focuses on promoting critical thinking, problem-solving skills, and creativity among students, and aims to foster a love for mathematics among young people. Through this initiative, we hope to create a more equitable and prosperous society by providing students with the skills they need to succeed in the 21st century.

2.2 The Kenya Math Camp

2.2.1 Introduction

The Kenya Maths Camp held in December 2022 was designed to promote learning of mathematics in a fun and engaging way. The camp was aimed at secondary school students, and the goal was to inspire them to view mathematics as an exciting subject and help them realize their potential to learn anything. Moreover, by using computers and introducing students to new themes such as robotics and statistics, they were exposed to cutting-edge technologies and emerging fields that could potentially lead to future career paths. The camp also provided opportunities for teachers to develop their professional skills and learn new teaching techniques. This section presents a detailed account of the themes and topics covered during the math camp.

2.2.2 Objectives

The objectives of the math camp were as follows:

- To provide secondary school students with a fun and interactive learning experience that would help them develop their math skills and knowledge.
- To introduce the students to a variety of math and computer related topics and inspire them to pursue further studies in STEM fields, with a particular focus on girls perspective.
- To create a supportive and inclusive environment where all students could feel comfortable asking questions and participating in activities, regardless of gender, age, socio-economic status and disabilities.
- To foster teamwork and collaboration among the students, and encourage them to share their ideas and insights with one another.
- To provide professional development opportunities for teachers by creating new didactic resources and by enhancing their pedagogical knowledge and developing new skills.

Figure 2.2: December 2022 maths camp participants and volunteers posing in front of the Kongoni Community Library.

2.2.3 Structure of the Math Camp

The camp was held in the Kongoni Community Library (KCL), Kitale, with the help and support of Supporting African Maths Initiative (SAMI), African Maths Initiative (AMI), INNODEMS, IDEMS. A total of 14 volunteers participated in the camp, consisting of 7 international volunteers and 7 local volunteers. 52 students aged between 14 and 19 years joined the camp from 29 different schools around Kitale area, with 73% of the students being female.

The camp was structured in a way that maximizes the students' learning and engagement: the days were divided into morning and afternoon sessions, with a break in between. The morning sessions were usually devoted to lectures, while the afternoons were dedicated to problem-solving sessions, where students work in groups to solve mathematical problems.

During the camp, students, participants and facilitators were divided into four houses, which were distinguished with different colours and were inspired by great mathematicians: Katherine Johnson (Orange), Leonhard Euler (Green), Sophie Germain (Blue) and Blaise Pascal (Red). Dividing participants into houses led to more efficient and enjoyable interaction. Collaborating with others enabled them to combine their thoughts and examine issues from diverse perspectives. In competitive scenarios such as physical activities and treasure hunts, house members employed their diverse skill sets and expertise to devise strategies and resolve challenges. Moreover, working in houses facilitated in-depth and comprehensive research, while also offering a platform for discussing, gaming, playing, and exploring ideas and questions and obtaining input from peers.

2.2.4 Planning and Preparation

The planning and preparation of the math camp was a crucial process that sets the stage for a successful learning experience for all involved. This phase involved a collaborative effort to share ideas, knowledge, and teaching methods from all the 14 volunteers and took place at the Kongoni Community library in Kitale. During the week-long preparation period, we worked together to plan the entire next week of the camp. This involved dividing different sessions by their themes, and organizing all the extra activities, such as the treasure hunt. To ensure that all volunteers were on the same page, a shared folder to share ideas and resources was created, and all the didactic materials necessary for the camp prepared. These materials will be also available and exploitable for future math camps and activities. The planning and preparation process allowed us to create a comprehensive curriculum that addressed the different levels and needs of the students who participated in the camp.
Overall, the week-long planning and preparation process for the math camp was crucial in ensuring that the camp ran smoothly and that the students were able to gain maximum benefit from the experience. It allowed us to organize and streamline our efforts, ensuring that all aspects of the camp were well thought out and executed.

2.2.5 Themes

The math camp covered a wide range of mathematical topics, including calculus, linear algebra, probability theory, and statistics. In particular, six main themes were explored: programming and robotics, modelling, mathematical thinking, data science, physical activities, and games and puzzles. In the following, a more detailed account of the themes and topics that were covered during the math camp will be provided.

1. Programming and robotics

The programming and robotics module of the math camp was designed to enhance students' logical and linguistic skills, foster project-based learning, problem-solving, gamification, and cooperative learning. The module engaged students in computational thinking, a process that involves planning, problem-solving, and analyzing information in a way that mimics a computer's logic. This approach enabled students to logically think through problems by breaking them down into smaller components, identifying patterns, and using their knowledge to develop step-by-step solutions.
The main objective of the module was to introduce students to the Scratch programming language through interactive human-robot games, Blockly games, and programming using Edison robots. Students learned about the capabilities of Edison robots through barcode programming, programming using blocks (Edblock), and Scratch programming (Edscratch), see picture 2.3 of students playing with these tools. Scratch is a free and open-source programming tool that offers a simple and enjoyable way for a broad range of users to learn, interact with, and create digital content. By introducing students to Scratch, they could continue working on their projects without losing the skills they had learned while also exploring a new way to engage with robotics.

Figure 2.3: Students playing with EdScratch and Edison robot.

During the module, students were guided through the process of creating a shark attack game using Scratch. This project provided a hands-on experience that allowed students to apply the skills they had acquired to create a functional and interactive game. The combination of hands-on activities and project-based learning helped students develop a deeper understanding of programming and robotics and fostered their creativity and problem-solving abilities.

2. Modelling

The ability to model mathematical activities and solve problems is a useful skill, both in classroom and in everyday life. One of the camp's themes was Modelling, which aimed to instil deep mathematical modelling skills in which students were encouraged to work and think, this included interactive processes that demanded creativity and inventiveness of the students in which mathematical, scientific and technical knowledge was applied to describe new situations. Throughout this theme, students were required to determine strategies, analysis or getting to the bottom of the problem provided and deploying Mathematical and computational tools. Modelling sessions involved physical activities and playing experimental games incorporating data collection, analyzing, recording, visualization and presentation where students were actively and directly involved. Students worked in groups to come up with the statistical information and ideas for the Data collection session, used that information to present data using Geogebra and Excel.

3. Mathematical Thinking

Mathematical thinking was a central theme of the camp, with the aim of developing participants' problem-solving and critical thinking abilities. Six sessions were planned to be carried out, each focusing on a specific topic, including factors and multiples, world cup combinatorics, divisibility rules, barcode tricks, Olympic rings, and Venn diagrams. These sessions were designed to instill in participants a solid understanding of mathematics, a methodical approach to thinking, and a high degree of organization. By using interactive and engaging activities, such as games and puzzles, participants were able to identify patterns and apply mathematical concepts to solve problems in a fun and challenging way. Through these sessions, participants learned to approach mathematical problems logically and systematically, developing skills that would be useful in their future studies and careers.

4. Data Science

Data science is becoming an essential skill in various industries and everyday life. As such, one of the themes that we focused on in the math camp was data science, with the aim of giving participants the chance to work with data and to view data science as both interesting and feasible. The five sessions that were dedicated to this theme aimed to teach participants how to understand the world through data exploration. Students worked with different data sets, summarised, visualized and interpreted data, thus enhancing their skills to use data as evidence to support their claims. The sessions were designed to give students the tools and confidence they needed to approach data in a meaningful way, and to develop an appreciation for how data can be used to solve problems and make decisions.

Figure 2.4: Students during data science sessions.

5. Physical Activities

The Physical Activities theme at the math camp aimed to provide participants with a unique opportunity to explore fundamental mathematical concepts through engaging in physical activities. The six different activities, including Trading Game, Concentration Game, Flag Game, Mingle Mingle, Find the Ball, and Treasure Hunt, were designed to be fun and engaging while simultaneously reinforcing mathematical concepts. Participants were able to practice counting, sequencing, one-to-one correspondence, number combinations, patterns, and computation strategies while playing the games.
In addition, the activities encouraged teamwork, cooperation, and communication skills, all of which are important for success in mathematics and in life. Participants were able to work in groups to solve problems and complete challenges, allowing them to practice critical thinking and problem-solving skills in a fun and interactive way. Through the Physical Activities theme, participants were able to see that mathematics can be fun and exciting and that it can be applied in many different contexts, including in physical activities.

6. Games and Puzzles

The games and puzzles sessions at the math camp were designed to provide the students with an opportunity to develop their problem-solving skills and explore various mathematical concepts. The activities covered a wide range of areas including logic puzzles, time distance maths, brain teasers, probability concepts, and number challenges. The puzzles were placed on the walls within the learning library section of the camp, allowing the students to solve them in their free time, and they were also given as take-home assignments. Although some of the puzzles were challenging, the students enjoyed the process of discovery and solving them. Each morning before the camp's normal sessions began, the challenging puzzles were solved as a class. The games and puzzles sessions also gave the students the opportunity to explore problem-solving concepts and take on the role of facilitators. The students were not only problem solvers, but they also supported others in engaging with the games and puzzles by ensuring that they understood the puzzles, asking questions for clarification, breaking down the puzzles into smaller parts, and reframing them. The puzzles were solved in groups, with each group having a different student as facilitator for different puzzle. The facilitators were able to actively tell stories and create activities that related the mathematical ideas to spark the memory of the other participants in the group. Through this, the students were able to develop their facilitation skills and learn how to support others in engaging with mathematical concepts.

2.2.6 Evaluation and Feedback

To assess the effectiveness of the math camp, two surveys were conducted: one on the first day and another on the last day. The first survey aimed to understand the students' expectations, career aspirations, and prior experiences with mathematics. The second survey aimed to gather feedback on the camp, including the effectiveness of the activities and the overall learning experience.

In addition to the surveys, students were encouraged to provide feedback through anonymous journal entries. These journals were collected at the end of each day and were helpful in identifying what worked well and what could be improved for future math camps. Furthermore, on the last day of the camp, several teachers of the participating students reported that this year's national examinations required critical thinking to solve problems, which was an advantage to previous camp participants who were sitting for the same exams. They also admitted the positive impact of the camp while they sat their examinations. This feedback was encouraging and suggested that the math camp was effective in promoting and improving students' mathematical skills and knowledge. Moreover, these findings show the potential for the math camp initiative to continue providing support and benefits to students beyond the camp itself.

The students' feedback and journal entries revealed the following benefits of the math camp:

- Increased confidence in their math abilities, as they were able to solve challenging problems and explore new concepts in a supportive environment, see for example the journal in Figure 2.5a: "I was happy and I gained some confidence answering questions".

- Greater commitment in scientific subjects, as the camp provided hands-on experiences and opportunities to apply mathematical concepts to real-world situations, making the students gain new skills and engagement with math. In journal reported in Figure 2.5b a student wrote: "The activities that we did yesterday were amazing and also made me enjoy and learn more. It also made me improve in my skills".

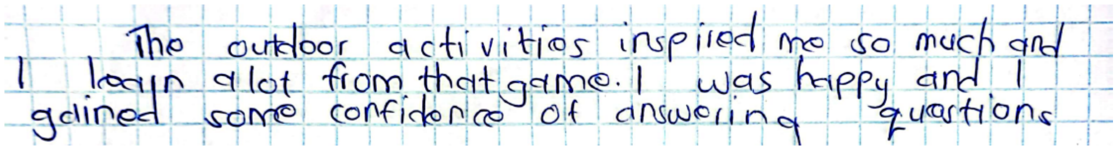
- Exposure to new topics, such as computer science, statistics and robotics, which helped broaden their understanding of the applications of math in different fields. Journal in Figure 2.5c reports: "Today was a really great day. I really enjoyed learning and creating different kinds of graphs ...".

- Improved teamwork and collaboration skills, as they worked in groups to solve problems and complete tasks (see Figure A.4). Moreover, it was an opportunity to network and make connections with like-minded peers, as the camp brought together students from different schools and backgrounds who share a passion for math.

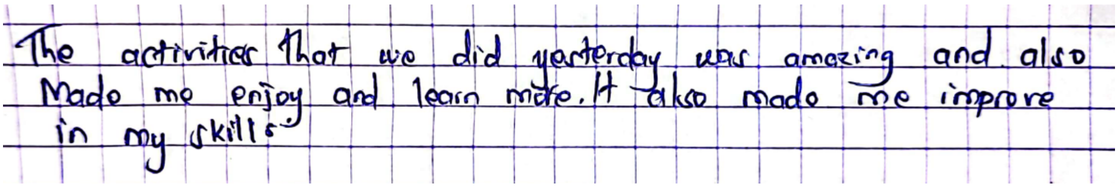
- Development of critical thinking and problem-solving skills through the challenging activities and problem-solving exercises provided at the camp.

- Enhanced communication skills, as students had to articulate their ideas and explanations to their peers during group work and presentations.

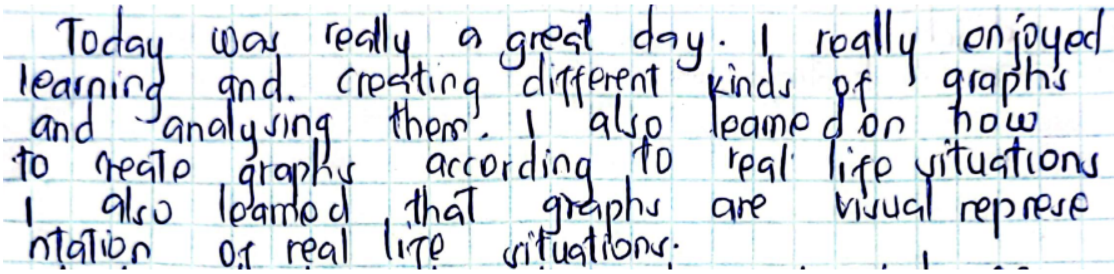
- Increased motivation to pursue further studies in math-related fields, as the camp introduced them to new and exciting areas of math and demonstrated the importance and relevance of math in various applications.


The outdoor activities inspired me so much and
I learn a lot from that game. I was happy and I
gained some confidence of answering questions

(a)


The activities that we did yesterday was amazing and also
Made me enjoy and learn more. It also made me improve
in my skills

(b)


Today was really a great day. I really enjoyed
learning and creating different kinds of graphs
and analysing them. I also learned on how
to create graphs according to real life situations
I also learned that graphs are visual represe
ntation of real life situations.

(c)

Figure 2.5: Example of journals with feedback from the students.

2.2.7 Next steps

The next steps for the math camp initiative include scaling the impact through the establishment of math clubs in schools. Math clubs are a weekly program designed for primary schools, secondary schools, and universities throughout Kenya to engage learners in mathematical puzzles, games, and extra-curricular activities with an emphasis on problem-solving, logic, and critical thinking. The main goal of these clubs is to shift students' negative attitudes towards mathematics. This will be done in tandem with the math camp to provide continuous learning and engagement opportunities for students. A total of 52 potential math game ambassadors from 29 schools were released back to their schools to introduce and play math games with whoever they have access to, including siblings, friends, classmates, and family members.

During the closing of the math camp, Debora Nyaboke, one of the trained ambassador from Friends School Kongoni Secondary School, who is currently sitting for her national exams, shared her testimony. Having attended the math camp in October 2021 and March/April 2022, in addition to being one of the 30 maths ambassadors trained in February 2022, improved her problem-solving, communication and presentation, critical and logical thinking skills. Debora shared that she is looking forward to facilitating sessions at the next camp and inspiring other students to become champions and ambassadors of maths games. Her example shows the way to go: building on the successes of the math camp by continuing to empower students to become ambassadors and champions of maths games and leveraging

their skills to benefit their communities.
Another important next step is to expand the reach of the math camp beyond Kenya. In December 2022, the Kenya Maths Camp brought together international volunteers from six different countries, including education officials and policy makers with a keen interest in running math camps in South Sudan and The Gambia in early 2023. This presents an exciting opportunity to share knowledge and resources and to replicate the success of the Kenyan math camps in other African countries. Through these new collaborations, students in South Sudan and The Gambia will have access to similar opportunities to improve their math skills, increase their confidence, and explore new topics like computer science and robotics. The experience of running these camps will also provide valuable insights and feedback that can be used to further improve the Maths Camp in the future.

2.3 Example of mathematical activity: The Josephus Problem

Overview

In this session, we will explore the Josephus problem, a classic problem in mathematics and computer science. The problem involves a group of children playing by forming a circle and removing themselves from the selection until only one person remains. We will initially demonstrate the problem and then work in groups to explore which position wins different cases of the game. We will identify patterns in the sequence of solutions and relate these observations back to the original problem. The session will encourage students to use inductive and iterative methods and to identify patterns in sequences.

Timing

The session will take 50-75 minutes (without the extensions).

Materials Required

- Students: pen and paper for working.
- Facilitator: whiteboard/blackboard to present assignment and solutions to the class.

Session Outline

The facilitator should begin the session by providing a demonstration of the problem to the group of students. One effective way to do this is to engage the entire group in the process, by showing the Josephus problem as a game. The game itself involves a group of children who must decide on a seeker for a game of hide and seek. To do this, they form a circle and follow a process whereby each child taps the shoulder of the child to their left, who then leaves the circle. This continues until there is only one child left, who becomes the seeker.

For example, in the case of $n = 5$, see Figure 2.6:

- Child 1 taps child 2, so child 2 is removed.
- Child 3 taps child 4, so child 4 is removed.
- Child 5 taps child 1, so child 1 is removed.
- Child 3 taps child 5, so child 5 is removed.
- So, child 3 is seeker.

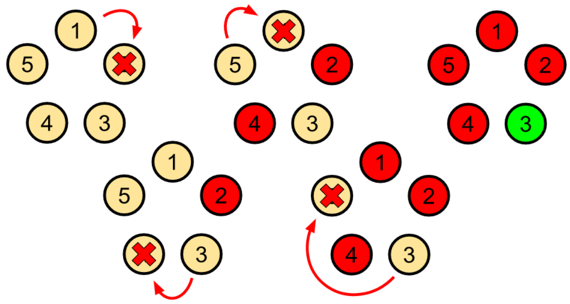


Figure 2.6: Illustration of the Josephhus Problem.

The facilitator can add an element of entertainment by intentionally placing himself in the winning position, allowing the students to recognize that there is an underlying strategy behind the game.

Considering cases and the general pattern

After the initial explanation, the objective is to determine the optimal position for winning the game within a circle of any given number of individuals. The players can be labeled with consecutive numbers starting from "player 1" who initiates the game. To initiate this inquiry, students should form teams of varying sizes and conduct experiments to determine the winning position for different scenarios of the game.
The results obtained can be compiled into a tabular format, with a preference for collecting data for games consisting of approximately 16-20 players. It is expected that the collected data will demonstrate a discernible pattern.
The students should be encouraged to explore any patterns in this sequence of solutions. Some prompting questions might be the following:

- Can you predict the next 5 winning positions for 21-25?
- What is the next number of players for which the winning position will be player 1?
- When does the player to the right of the first player win?

Number of players	Winning position
1	1
2	1
3	3
4	1
5	3
6	5
7	7
8	1
9	3
10	5

Number of players	Winning position
11	7
12	9
13	11
14	13
15	15
16	1
17	3
18	5
19	7
20	9

If the students manage to identify any patterns they should be encouraged to come to the board to demonstrate to the class. Some examples of patterns that may be seen are:

- The winning position is always odd.
- The sequence of winning positions tends to increase by 2 each time (except in some cases where it goes back to 1).
- Whenever the number of players is a power of 2, the winning position is 1.
- The last player (on the right of the first player) wins whenever the number of players is one less than a power of 2. After this, the next winning position is always 1.

Knowing this, the students can predict the next winning positions:

Number of players	Winning position
21	11
22	13
23	15
24	17
25	19
26	21
27	23

Number of players	Winning position
28	25
29	27
30	29
31	31
32	1
33	3
34	5

Relating observations to the original problem

The students can now analyze the relationship between these observations and the original problem, with the goal of determining the optimal winning position for any number of players. Our approach involves first examining the case when the number of players is a power of two, and demonstrating that this will always result in player 1 as the winner. Subsequently, we can extend this analysis to cases where there are any number of players, by relating it to the game played with one fewer player.

Number of players is a power of two - Questions: We begin the game with 16 players, how many players are left when the turn comes back to player 1? Can you explain why player 1 is the winner both when there are 8 and 16 players? How about when there are 32 players, 64 players, 128 players, . . . ? Do you notice a pattern?

When the number of players is a power of two, we can observe that playing the game once around the circle will half the number of players and we will begin again at player 1. For example, taking 16 players initially:

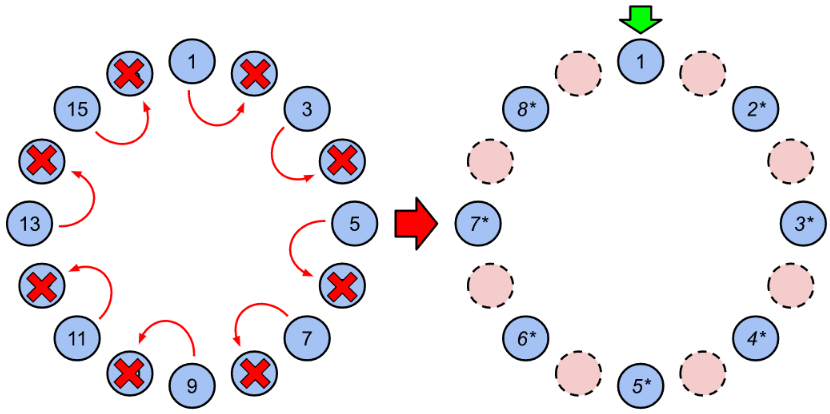


Figure 2.7: Illustration of the Josephus Problem with 16 players.

After one full round we have reduced the number of players to 8 and player number 1 begins again. So if player number 1 wins in the 8 player game, then player 1 must also win in the 16 player game!
Similarly we can see in the 32 player game that after one round the game is reduced to the 16 player case, and from 64 we reduce to the 32 case, etc. So for any power of two, the winner is going to be player 1. We have verified that this is true in the first few cases.

Relating to the previous cases - Questions: We begin the game with 17 players, how many are left after 1 turn? Who is going to win the game? What about when we begin with 33 players, 65 players, 129 players, . . . ? Do you notice a pattern? We begin the game with 18 players, who will win the game? How about 19, 20, 21, . . . players? If we know that the number of players is n more than a power of two, which player will win?

Say now that we have a number of people that is one more than a power of two. We notice that after the first child puts the second out of the game, the number of players that we have left is a power of two.

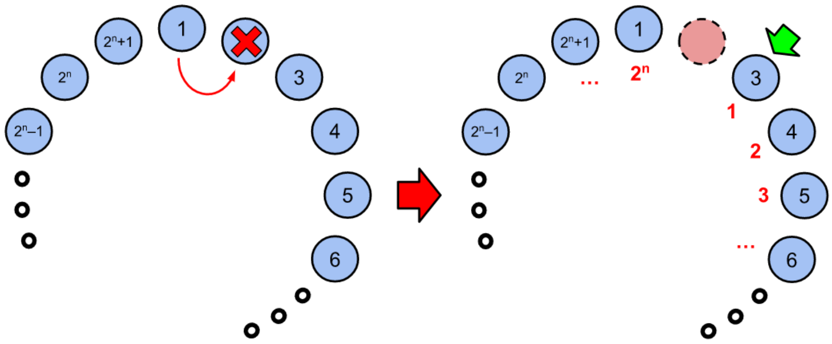


Figure 2.8: Illustration of the Josephus Problem with a different number of players.

So knowing that the first player wins when the number of players is a power of two. We can say that the third player is going to win in this case.
Similarly, if we have two more players than a power of two the fifth player is going to win because after two moves the number of players left is a power of two and the fifth player is the next to move.
We can therefore explain the behaviour that we see in the pattern where the winning player for each number of players increases by two each time, and resets to the first player at every power of two.

Extensions

There are several possible extensions for this session about the Josephus problem:

- Explore variations of the problem: The basic Josephus problem assumes that every kth person is eliminated in a circle of n people. This rule can be modified by, for example, eliminating every other person, or eliminating every third person in a clockwise direction. Students can be asked to analyze how the winning position changes for different variations of the problem.
- Connect the Josephus problem to other mathematical concepts: The Josephus problem is related to a variety of mathematical concepts, such as modular arithmetic, recurrence relations, and graph theory. Some time can be spent exploring how these concepts are related to the problem, and how they can be used to solve related problems.
- Apply the Josephus problem to real-world scenarios: The Josephus problem can be used to model a variety of real-world scenarios, such as voting systems, military tactics, and network protocols. Students can explore how the problem can be applied to these scenarios, and how the insights gained from the problem can be used to optimize the systems.

Chapter 3

Mathematics Education in South Sudan

South Sudan is a country facing significant challenges in providing education to its population, particularly in mathematics. The country faces disparities in educational access and outcomes based on factors such as gender, ethnicity, socioeconomic status, and location. The achievement of Sustainable Development Goal 4, which aims to ensure inclusive and equitable quality education for all, is therefore a critical goal for the country's development.

South Sudan is one of the youngest countries in the world, having gained independence from Sudan in 2011. Despite efforts to expand education access in the years since, the country still faces significant challenges. Here are some data that illustrate the challenges facing education in South Sudan:

- **Enrollment Rates**: According to the UNESCO Institute for Statistics, the primary net enrollment rate in South Sudan was 44% in 2019, compared to a global average of 90%. The secondary net enrollment rate was only 6%, compared to a global average of 84%. This is a significant concern given the strong correlation between education and economic and social development. Without access to education, children in South Sudan are at a disadvantage when it comes to finding jobs, improving their standard of living, and contributing to the development of their country.

- **Gender Disparities**: Girls in South Sudan face significant barriers to accessing education. The primary net enrollment rate for girls was only 36% in 2019, compared to 52% for boys. The gender gap is even wider at the secondary level, where the net enrollment rate for girls was only 4%, compared to 9% for boys, UNESCO Institute for Statistics (2019). This disparity is likely due to a combination of factors, including cultural norms, poverty, and the lack of infrastructure and resources to support girls' education.

- **Completion Rates**: Completion rates for primary school in South Sudan are also low. In 2019, only 27% of primary school-age children completed primary school, compared to a global average of 83%. The completion rate for lower secondary school (grades 7-9) was only 5%. The low completion rates in South Sudan suggest that many children who do enroll in school are not able to finish their education. This may be due to a variety of factors, including poverty, conflict, and the lack of resources and qualified teachers.

- **Teacher Shortage**: South Sudan faces a severe shortage of qualified teachers, particularly in rural areas. According to the World Bank, there are only 4 teachers per 1000 primary school students in South Sudan, compared to a regional average of 17. The shortage is even more acute at the secondary level, where there are only 2 teachers per 1000 students (The World Bank, 2021). This results in very high teacher-pupil ratios, with an average of 90 pupils per teacher in primary schools and 40 pupils per teacher in secondary schools. The shortage of qualified teachers is a major barrier to improving the quality of education in South Sudan. Without enough trained and skilled teachers, it will be difficult to provide high-quality education to all children, particularly those in rural areas.

- **Infrastructure**: Many schools in South Sudan lack basic infrastructure, such as classrooms, desks, and textbooks. In 2019, only 10% of primary schools had access to basic sanitation facilities, such as toilets, and only 11% had access to electricity.

- **Conflict**: The ongoing conflict in South Sudan has disrupted education access for millions of children. Since the beginning of the war in 2013, over 150 schools were utilized for military purposes and numerous children were abducted from their classrooms. The constant violence and threat of violence left 2 million children out of school in 2017, and hundreds of schools and other civilian assets were pillaged and destroyed, UNICEF.

Overall, the data on education in South Sudan paint a concerning picture of access and quality. Addressing these challenges will require significant investment and a coordinated effort from government, international organizations, and civil society. However, improving education in South Sudan will be essential to building a brighter future for the country and its people.

The Role of Catholic Church

The Catholic Church has played an important role in promoting education and peace in South Sudan. Since the country gained independence in 2011, the Church has worked to establish schools and educational programs, particularly in areas with limited access to education. During the decades of conflict and instability in South Sudan, the Catholic Church remained a key provider of education services, even in some of the most remote and challenging parts of the country. The Church established schools and other educational programs in areas where there were no other providers of education, and it worked to ensure that children in these areas had access to quality education. According to the Catholic Relief Services, as of 2020, the Catholic Church runs approximately 5000 schools in South Sudan, serving more than 2 million students. These schools provide education to children and young adults in some of the most remote and conflict-affected areas of the country, where access to education is limited. In addition to providing education, the Church has also focused on promoting peace and reconciliation through its pastoral work, interfaith dialogue, and efforts to address the root causes of conflict. The Church has been a trusted mediator in peace negotiations, and has played a key role in promoting national reconciliation and healing in the aftermath of the country's civil war. Through its work, the Church in South Sudan continues to play an active role in building a more peaceful and prosperous future for the country.

3.1 Mathematics initiatives in South Sudan

Maths initiatives in South Sudan have been a growing focus in recent years, as the country works to build a stronger education system and address the challenges faced by students and teachers. Non-governmental organizations (NGOs), such as Save the Children and UNICEF, have implemented programs to support maths education in South Sudan, particularly in areas affected by conflict and displacement. These initiatives aim to improve maths skills among students, increase the number of trained maths teachers, and provide educational resources to schools. For example, Save the Children has implemented a programme to support teacher training in maths, which has helped to improve teaching methods and increase student engagement. UNICEF has also established numeracy centers in areas affected by conflict, providing students with access to resources and support to improve their maths skills.

Additionally, the South Sudanese government has made efforts to strengthen maths education in the country, including prioritizing maths curriculum development and teacher training. The government recognizes the importance of maths education in building a stronger education system and providing students with the skills they need to succeed in the future. To this end, the government has developed a national maths curriculum and provided training opportunities for teachers to improve their maths teaching skills.

Despite progress, however, significant challenges remain. Many schools in South Sudan lack resources and infrastructure, making it difficult to provide quality education to students. Ongoing conflict and instability in some regions have also disrupted education and made it difficult for students and teachers to access resources and support. Moreover, the COVID-19 pandemic has had a significant impact on education in South Sudan. Schools were closed in March 2020 to curb the spread of the virus, and many have not reopened for more than a year. According to the Ministry of General Education and Instruction, as of May 2021, only 16% of schools had reopened after the initial closure in March 2020. This has disrupted learning for millions of students, particularly those from low-income families and rural areas who lack access to digital resources. School closures have had a negative impact on the mental health and well-being of students, particularly girls who face a higher risk of early marriage and dropping out of school. The pandemic has also led to a decrease in funding for education, from 9.7% government's expenditure in 2018 to 7.5% in 2020, as resources are redirected to COVID-19 response efforts. As a result, the education system in South Sudan has been severely impacted, and the long-term effects of the pandemic on education remain uncertain (UNICEF Eastern and Southern Africa Regional Office, 2021).

Nonetheless, the focus on maths education in South Sudan is an important step in building a stronger education system and providing students with the skills they need to succeed in the future. NGOs, the Catholic Church and the government must continue to work together to overcome the challenges and ensure that all students have access to quality maths education.

3.2 My experience teaching mathematics in South Sudan

3.2.1 Introduction

From mid-January to the end of February, I had the privilege of working in several schools in South Sudan, specifically in Rumbek, Lakes State. During this time, I employed various teaching methods such as games and ethnomathematics to engage both secondary school students and primary school teachers and to foster a greater interest in mathematics and scientific subjects by presenting an innovative and engaging approach to learning. To measure the effectiveness of these activities, I collected feedback from the participants and found that they were positively received, resulting in a significant improvement in students' engagement with mathematics. In particular, I focused on promoting girls' education in South Sudan. Girls in South Sudan face significant barriers to education, including poverty, early marriage, cultural attitudes, and limited access to schools. According to UNICEF, only 17% of girls in South Sudan complete primary school, and only 3% go on to secondary school. In addition to my teaching role, my visits to these schools enabled me to gain insights into the other challenges faced by educators in providing quality education to their students. These challenges include limited resources and infrastructure, lack of qualified teachers, and the impact of conflict on schools.

I would like to express my sincere gratitude to the Fondazione CESAR Onlus and the Dioceses of Rumbek for their generous support of my work in South Sudan and for the continuous work they are doing to support education in the country. Their commitment in the region is truly inspiring, and I am honored to have had the opportunity to work with them.

3.2.2 Background

To provide context for my teaching experience, I will begin by discussing the history and culture of Lakes state in South Sudan, including the role of education in the region. Lakes state is situated in the central part of South Sudan and is home to several ethnic groups, including the Dinka, Nuer, and Shilluk. Historically, education has played a significant role in the region, with mission schools established by various Christian denominations during the colonial period. These schools were often the only source of education for local children, and they provided not only academic instruction but also religious and moral guidance. Following Sudan's independence in 1956, the government took over the administration of education, but resources were limited, and many schools were poorly equipped and understaffed. The ongoing conflict and instability in South Sudan have

further disrupted the education system, with many schools damaged or destroyed and teachers forced to flee their homes. Despite these challenges, education remains a top priority for the government, local communities and the Catholic Church, and efforts are underway to rebuild and strengthen the education system.
During my time in Lakes state, I had the opportunity to visit several schools, each with its own unique history and context. In the following section, I will provide an overview of these schools and the work I carried out with them.

Mazzolari Secondary School

Mazzolari Secondary School is a co-educational secondary school located in Rumbek, the capital city of Lakes state in South Sudan. The school was founded in 2010 by the Comboni Missionaries, an Italian Catholic missionary congregation, with the goal of providing education to children and young people in the region. The school serves over 400 students from grades 7 to 12, with a focus on academic excellence and Christian values. The curriculum includes subjects such as math, English, science, and social studies, as well as religious education.
Like many schools in South Sudan, Mazzolari Secondary School faces several challenges, including limited resources and infrastructure. The school has few classrooms and lacks basic amenities such as electricity and running water. Teachers also face difficulties in attracting and retaining students, particularly girls, due to cultural barriers and the high cost of education. Despite these challenges, Mazzolari School has achieved notable successes, with several students going on to pursue higher education at universities both within and outside of South Sudan. Mazzolari Secondary School has also been involved in several community development projects, including the construction of a water well and the establishment of a vocational training center for women. These initiatives reflect the school's commitment to not only providing quality education but also improving the lives of those in the local community.

Figure 3.1: Class presentation at Mazzolari Secondary School, Rumbek.

Loreto Primary and Girls Secondary School

Loreto Primary and Girls Secondary School is a Catholic school located in Rumbek, the capital city of Lakes state in South Sudan. The school was established by the Loreto Sisters in 2008, with the aim of providing education to girls in the region. The school currently has around 1200 students from grades 1 to 12, with separate facilities for primary and secondary students. The Loreto Girls Secondary School has approximately 400 students from grades 9 to 12. The school is committed to providing a high-quality education to its students, with a focus on academic excellence, leadership development, and community service.

One notable achievement of the Girls Secondary School is its success in promoting girls' education in a region where girls often face significant barriers to accessing education. The school has a high retention rate, with the vast majority of students completing their studies and going on to pursue higher education or employment. Indeed, the School's dropout rates have remained under 5% since 2014. In addition, the school has a strong track record of academic success, with many students achieving high scores on national exams. Moreover, Loreto Girls Secondary School has implemented a beneficial internship program of one or two years for its graduating students. Upon completion of their secondary education, students are given the opportunity to participate in the program, which aims to provide valuable work experience and professional skills. As an added incentive, the school awards a scholarship to cover the costs of university expenses upon successful completion of the program. This initiative not only prepares students for their future careers but also encourages them to pursue higher education by easing their financial burden.

Loreto is known for its commitment to promoting gender equality and empowering girls through education. The school has several initiatives aimed at increasing girls' access to education, including a scholarship program for disadvantaged students and a community outreach program that promotes the importance of education for girls. In addition, the school has a strong focus on community service and social justice, with students participating in a range of activities to support the local community. The school also has a student council that provides leadership opportunities and helps to foster a sense of community among the student body.

During my time at Loreto Secondary School, I had the privilege of working with two classes of students: the Transition class and the Senior Four. Loreto is a girls-only boarding school that admits students based on their performance in Mathematics and English tests, as well as an oral English interview. Those who pass are admitted to Senior One, while promising students enter the Transition Year class. The Transition Year serves as a preparatory class, helping students to strengthen their academic foundations. At the end of the year, students are assessed, and those who pass are promoted to Senior One.

Furthermore, I also conducted a one-week teacher training with seven primary school teachers, who were mainly youth that had completed their secondary education with little to no knowledge of education or pedagogy. During the training, I shared my approach of using games and ethnomathematics to increase engagement in math and scientific subjects. I showed them how to structure their lessons by starting with observation of a general problem and gradually diving deeper into a specific problem, always emphasizing the connection between the subject matter and real-life applications and problems. This training aimed to equip these teachers with new tools and methodologies to enhance their teaching practices and ultimately improve the quality of education provided to their students.

Figure 3.2: Students of the Transition Class in Loreto Girls Secondary School, Rumbek.

All Saints ALP School

All Saints ALP School is a primary school located in Cueibet, Lakes State, South Sudan. It is a community school that was established in 2014 with the aim of providing education to children who had been affected by conflict in the region. ALP stands for "Accelerated Learning Program": it is a type of educational program designed to provide basic literacy and numeracy skills to children and youth who have missed out on formal education due to conflict, displacement, or other factors. The program condenses the primary school curriculum into a shorter time frame, allowing students to catch up on missed years of schooling and transition into higher levels of education. Specifically, the program of All Saints ALP School in Cueibet condenses the eight-year primary school curriculum into four years of accelerated learning. Some students are of primary school age, while others are older teenagers or young adults.

One of the challenges facing All Saints ALP School is a lack of resources and infrastructure. The school operates in a temporary facility and lacks basic amenities such as running water and electricity. In addition, many students come from low-income families and struggle to afford basic school supplies and uniforms. Despite these challenges, the school has had some notable achievements. For example, in 2020, All Saints ALP School received a grant from a local NGO to improve its infrastructure and purchase new teaching materials. In addition, the school has a high retention rate, with the vast majority of students completing their studies and transitioning to secondary education.

3.2.3 Methodology

During my time in the previously mentioned secondary schools in Rumbek, South Sudan, I implemented a methodology aimed at creating a stimulating environment for students and teachers alike. This methodology was centered around working in groups, using games and ethnomathematics to encourage exploration and inquiry. By working in groups, students were able to collaborate and learn from one another, sharing their ideas and approaches to problem-solving. Moreover, they were asked to come and play on the backboard, or to explain their solving methods to the whole class. In this way, they could gain confidence and enhance their skills in presenting results and supporting assumptions.

Figure 3.3: Students playing a game on the blackboard after the explanation.

In addition, the use of games and ethnomathematics helped to make the subject more accessible and enjoyable for students who may have had limited exposure to math before. For example, by incorporating traditional African fractals into my teaching, I was able to connect algebra and geometry concepts to cultural practices and real-life situations. By using this approach, I aimed to improve students' understanding and motivation in mathematics while contributing to the development of a more culturally responsive and inclusive approach to education. The implementation of this methodology is expected to have several benefits, including improved academic performance, increased engagement, and a more positive attitude towards mathematics.

Overall, this approach to mathematics education highlights the potential of incorporating cultural practices and students' backgrounds into the learning process. By doing so, we can create a more equitable and effective learning environment for all students, regardless of their social, economic, or cultural background.

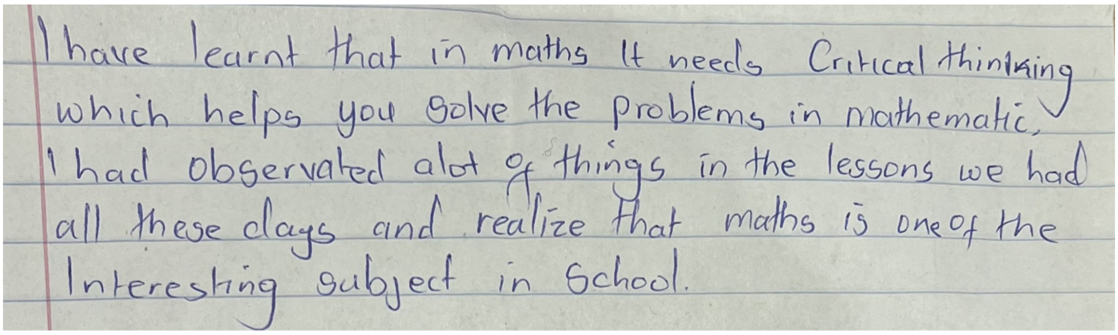
3.2.4 Evaluation and Feedback

To enhance the teaching and learning process and understand the impact of innovative teaching methods, feedback was sought from both students and teachers after each mathematical activity. Anonymous surveys were used to collect this feedback, which was distributed and collected by the researcher. The collected data was then analyzed to identify areas of improvement and inform necessary adjustments to the mathematical activities, ensuring they were better aligned with the needs and expectations of the students. This feedback was also crucial in understanding whether the students' engagement was enhanced through the use of innovative teaching methods such as games and etnomathematics.

Through the feedback received from both students and teachers, it became apparent that many of the students were accustomed to rote learning and memorizing rules in math, rather than engaging in critical thinking and problem-solving. Instead, in the activities conducted, the students were encouraged to approach mathematical problems using their own everyday experiences and cultural backgrounds, and to develop unique solutions based on their personal perspectives. This approach was novel for many of the students, and they reported feeling more engaged and motivated to participate in class. See for example the journal in Figure below where a student writes: "I have learnt that in maths it needs Critical thinking which helps you solve the problems in mathematic. I had observed a lot of things in the lessons we had all these days and realize that maths is one of the most interesting subject in school". By fostering a culture of creative problem-solving and encouraging students to think independently, I hoped to empower the students and cultivate a lifelong love of mathematics.

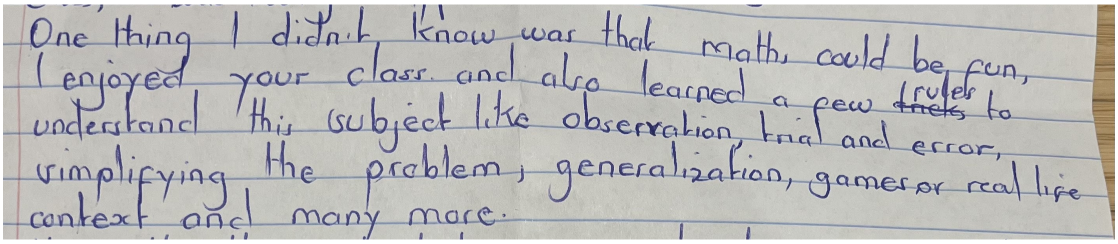
The interactive and collaborative nature of the activities, coupled with the emphasis on personal experiences and cultural backgrounds, proved to be a winning combination. Many students reported finding the sessions fun and engaging, and some even expressed a newfound interest in math as a result of the activities. Additionally, several students reported feeling more confident in their ability to approach math problems creatively and independently, rather than relying solely on memorized rules and formulas. See for example the comment in Figure here reported: "One thing I didn't know was that maths could be fun. I enjoyed your class and also learned a few rules to understand this subject like observation, trial and error, simplifying the problem, generalization, games or real like context and many more".

Moreover, many students have given positive feedback on their experience with ethnomathematics activities. Students have reported that participating in these activities has



I have learnt that in maths It needs Critical thinking which helps you solve the problems in mathematic, I had observated alot of things in the lessons we had all these days and realize that maths is one of the Interesting subject in School.

Figure 3.4: Feedback from a students after the activity.



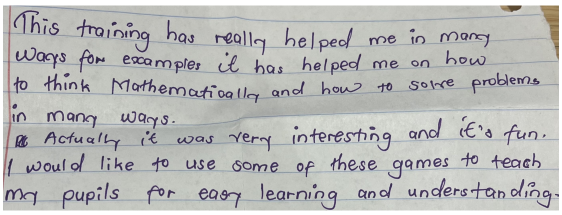
One thing I didn't know was that maths could be fun, I enjoyed your class and also learned a few ~~tricks~~ rules to understand this subject like observation, trial and error, simplifying the problem, generalization, games or real life context and many more.

Figure 3.5: Feedback no.2 from a student after the activity.

helped them see math in a new light and understand its relevance to different cultures and contexts (see Figure A.6). By learning about the mathematical practices and traditions of their cultures, students have gained a deeper appreciation for the subject and its applications. Additionally, ethnomathematics activities can help promote diversity and inclusivity in the classroom, as students from different backgrounds are able to share their perspectives and experiences with one another. Overall, students' feedback suggests that ethnomathematics activities had a positive impact on their attitudes towards math and their appreciation for cultural diversity.

All things considered, the feedback from the students suggests that the activities were successful in encouraging critical thinking and problem-solving skills, as well as increasing engagement and interest in the subject matter. The positive feedback from the students was echoed by the teachers who participated in the training.

Teachers reported a lot of interest in the activities and expressed enthusiasm for incorporating the ideas into their classroom teaching. By providing hands-on training and modeling innovative approaches to math education, I hoped to inspire more teachers to adopt similar methods and create a more engaging and effective learning environment for their students.



This training has really helped me in many ways for examples it has helped me on how to think Mathematically and how to solve problems in many ways.
Actually it was very interesting and it's fun. I would like to use some of these games to teach my pupils for easy learning and understanding.

Figure 3.6: Feedback from a teacher about the training.

The success of the activities and the positive feedback received suggest that there is great potential for these methods to be scaled up and implemented on a wider scale, ultimately benefiting students and educators alike.

3.2.5 Next steps

Overall, my experience in Lakes state has highlighted the potential of etnomathematics and innovative teaching methods to support math education in South Sudan. To further advance this work, a next step will be to organize a math camp in South Sudan, similar to the one successfully implemented in Kenya. This math camp would be a great opportunity to bring together teachers and students from different regions of the country to learn and share innovative teaching practices that incorporate etnomathematics. By doing so, we hope to promote a culture of innovation and continuous learning in math education in South Sudan.

Additionally, I will remain in contact with the teachers I worked with to gather feedback and evaluations about the activities they will develop following the teacher training. This feedback will help us identify areas for improvement and develop more effective teacher training programs in the future. It will also help us to monitor and evaluate the impact of our interventions and make necessary adjustments.

In addition, while I was in the University of Juba, I supported in the organization of the national phase to select a team of seven students, with at least three students from each gender, to participate in the East African Maths Olympiads, which will take place on April 20th in Kigali. While the selection was limited to schools near Juba, there is a need to expand this competition to schools in other regions, such as Rumbek. This will help us to identify and nurture math talent from all regions of South Sudan, and to promote competition and motivation among students.

My findings suggest that etnomathematics has the potential to promote cultural relevance and student engagement in math, but further research is needed to explore its effectiveness in diverse contexts. Therefore, we plan to conduct more research to assess the impact of our interventions in different regions of South Sudan. Ultimately, my hope is that my work can contribute to improving math education and promoting educational equity in South Sudan, and to inspire others to continue this work.

3.3 Example of mathematical activity: From fractals to complex numbers

Topic

This activity aims to explore the concept of fractals and the underlying mathematics behind them, with a focus on understanding their properties such as scaling and recursion, through the lens of geometry, nature, culture and algebra. Additionally, the activity will delve into the application of these concepts to complex numbers, in order to demonstrate the impressive results that can be achieved. The activity can be included in the field of etnomathematics, which is the study of the relationship between mathematics and culture, as it will utilize African culture as a starting point to showcase the applications of fractals in a culturally meaningful context. Through exploring some examples, students can gain an appreciation for the diverse ways in which fractals and mathematics have been incorporated into cultural expressions. Moreover, the activity incorporates group discussions and reflections on the cultural contexts that influence the creation and interpretation of fractals. This would encourage students to think critically about the role of culture in shaping mathematical practices and to consider how different cultural perspectives can contribute to the development of new mathematical ideas. By engaging in this activity, students can gain a deeper understanding of fractals and the mathematical concepts that underpin them, while also appreciating the rich and varied cultural contexts in which they arise.

Purpose and learning objectives

The purpose of this learning activity is to enable learners to recognise similarities and properties across different contexts and develop an understanding of the concepts of self-similarity, scaling, and iteration. Participants will apply these concepts in geometry to create simple to complex fractals. They will also be introduced to the concept of complex numbers and the complex plane, and understand the relationship between complex numbers, finite progressions, and the Mandelbrot set. Overall, the learning objectives of this activity are to equip learners with a deeper understanding of fractals and their properties, as well as the application of these concepts in different areas.

Prerequisites required for the students to carry out the activity

This activity is designed for secondary school students, and can be adapted to suit different levels of mathematical proficiency. For instance, students at lower levels could delve into a more in-depth analysis of plane or solid geometry during the Sierpinski session. On the other hand, more advanced students can explore complex arithmetic in greater detail, especially after the Mandelbrot session.

Class organisation and methodology

The activity was specifically designed to be implemented in secondary schools in South Sudan, where access to internet connectivity and computer devices is limited. As such, the methodology for this activity comprises a combination of lectures, graphical illustrations, and practical activities. The use of software and computers can enhance certain sessions or enable the exploration of more sophisticated fractals, see Extensions in the last session. Overall, the approach of the whole activity is intended to foster exploration, reflection, conjecture, and argumentation with the teacher. Through hands-on activities, students will have the opportunity to apply the concepts they have learned and work collaboratively to deepen their understanding of fractals and related mathematical concepts. During the stages of argumentation and collective discussion, the teacher serves primarily as a facilitator, enabling the class to object, rebut, provide examples and counterexamples, and ultimately reach a conclusion that leads them to the "discovery" of fractals.

Phases and timing (indicative)

The teaching proposal comprises four sessions with a total duration of five hours. To evaluate the extent to which the learning objectives have been achieved, a pre- and post-activity test may be administered. Furthermore, a survey could be conducted to assess the extent of students' engagement in exploring topics that connect mathematics with culture. This survey would be particularly useful in assessing the impact of ethnomathematics on classroom instruction.

Session	Time
The idea of fractals	1h
Fractals in African culture	1/1.5h
Fractals in Geometry - the Sierpiński Triangle	1h
Fractals in Algebra - the Mandelbrot Set	2h

Bibliography

The inspiration for this teaching activity originated from Ron Eglash's book *African Fractals*, which explores the fractal patterns underlying art, architecture, and design in various regions of Africa (Eglash, 1999). Eglash and his team also created the Culturally Situated Design Tool (CSTD), a computer software that enables students to design and modify traditional cultural patterns, including jewelry, hairstyles, and urban architecture. The use of the software was not included in the teaching activity, but it can be used as a useful tool for students to further explore and experiment with fractal patterns. Additionally, this activity is inspired by the work of the Fractal Foundation, whose mission is to promote interest in science, mathematics, and art through the beauty of fractals. The foundation's work serves as an inspiration and demonstrates the wide-ranging applications of fractals in various fields. In line with their mission, this teaching activity aims to encourage students to appreciate the beauty and usefulness of fractals in mathematics, science, and culture.

3.3.1 Session No.1: The idea of fractals

Overview

In this session, the concept of fractals will be introduced, with a focus on their main properties of self-similarity, scaling, and recursion. Students will be encouraged to reflect on these properties and explore how they can create their own fractals.

Timing

The session will take 1 hour.

Materials Required

- Students: pen and paper for working.
- Facilitator: whiteboard/blackboard to present assignment and solutions to the class; if available, computer software can be used to help students become familiar with fractals and create and test more complex patterns.

Session Outline

The facilitator should begin the session by providing an overview of fractals, which are patterns that repeats itself at different scales. They are highly effective in replicating the patterns found in nature, such as the branching of trees, the formation of mountains, and the structure of clouds. To inspire the students, the facilitator should display visual aids featuring examples of fractals in nature and ask the students to reflect on what they see. For instance, by looking at a picture of a tree, students will be able to identify some of these properties:

- a small segment of the tree bears resemblance to the whole tree.
- to draw a tree, you typically begin with a sprout emerging from the ground, which then bifurcates into branches; these branches then subdivide further into even more branches, and this process repeats itself multiple times.
- at some point, the repetition stops and the tree is complete, it does not continue forever.
- every time the tree divides into branches, they become thinner than the previous ones.

Following the discussion about the properties of fractals, the facilitator should present an image of a Koch curve, Figure 3.7, which is a type of fractal named after Swedish mathematician Helge von Koch. The facilitator should encourage the students to observe the curve and identify any similar properties to the tree discussed earlier. The facilitator should also prompt the students to think about the process that was used to draw the curve.

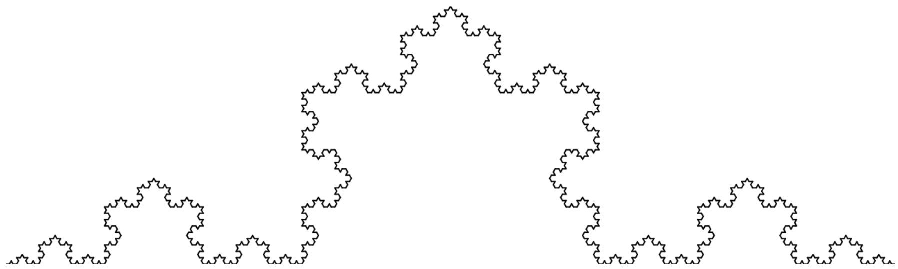


Figure 3.7: Example of a Koch curve

The facilitator will explain the following properties to the students:

1. Scaling: The Koch curve starts as a simple shape with just 4 lines. Then, using a scaling transformation, we shrink down the shape and replace each of the 4 lines with a miniature version of itself. This process is repeated for all 4 lines, resulting in the output of the first iteration.

2. Recursion: The same process is repeated in each iteration, where lines are replaced with miniature versions of the original shape, and the output of one iteration becomes the input for the next. This process is called a "recursive loop", where outputs are used as inputs.

To conclude the session, the facilitator should ask the students to apply what they have learned and create their own fractal on paper. The students should aim to create three or four iterations and reflect on what they want as the generator or original shape, and what scaling factor they will use. The students should be encouraged to share and discuss their creations with the class, and reflect on the similarities and differences between their fractals and those of their peers. Moreover, a computer software can be used to help students become familiar with fractals and create and test more complex patterns. Many open-source Fractal Software Generators are available online, as for example Apophysis, which can be downloaded from `https://sourceforge.net/projects/apophysis/`.

3.3.2 Session No.2: Fractals in African culture

Overview

In this session, students will delve more into the theme of fractals, by exploring African culture and architecture. Students will be encouraged to discuss and present their findings to the whole class. Moreover, through this session they will discover the universality of mathematical concepts and their importance in shaping human expression and creativity.

Timing

The session will take 1/1.5 hours minutes based on how many students are present in the class for the presentations.

Materials Required

- Students: pen and paper for working.
- Facilitator: whiteboard/blackboard to present assignment and solutions to the class; worksheet for every group with different examples of African fractal designs.

Session Outline

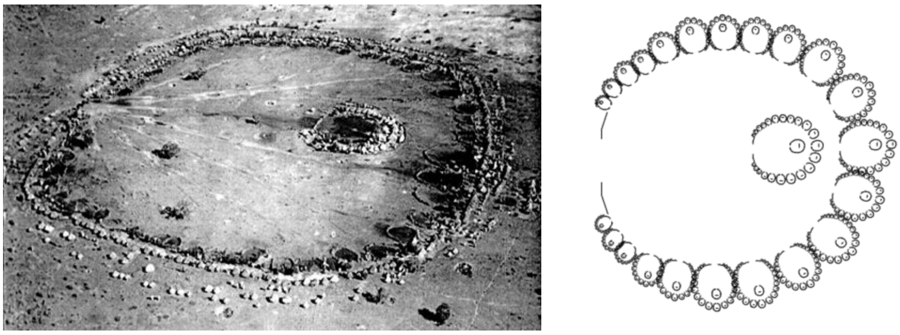
The facilitator should start introducing examples of African fractal designs: indeed, anthropologists have observed that many Indigenous African societies used fractal designs in their architecture, textiles, sculpture, art, and religion. This wasn't just a coincidence or intuition, as Africans were able to connect these designs to concepts like recursion and scaling. It's interesting to note that while other cultures may have focused on Euclidian geometry, Africans have placed a greater emphasis on fractal designs. This reflects their unique cultural priorities, which can even be seen in their music with its use of simultaneous rhythms at different scales.
The main part of the session will be developed in small groups (4-5 students), where students will analyze a specific example of an African fractal design provided by the facilitator. They should try to identify the generator and recreate the first iterations of the fractal using paper and pencil. In addition, they will research and discuss the cultural significance of the design and identify reasons why the African society in question might have chosen to use this particular structure. Finally, each group will present their findings to the class, sharing their discoveries on fractals in African culture. After every presentations, students should be encouraged to think of other fractals examples from their own culture. This activity aims to deepen their appreciation for the versatility and beauty of these mathematical concepts and enhance their skills in presenting results and supporting assumptions.

Example: the Ba-ila settlement

The facilitator will provide to a group of students these images of Ba-ila settlements (Figure 3.8) of Southern Zambia with some core information:

- The settlements are organized around a central courtyard or common space, made up of smaller rings, which are livestock pens (corrals). Those rings are made up of smaller rings which are single cylindrical houses and storage rooms. It is a ring of rings of rings.
- The arrangement of the houses is highly ordered, with each family having a specific location in the settlement;
- The circular design of the settlement is thought to reflect the Ba-ila's cosmology and worldview, which emphasizes the importance of harmony, balance, and interconnectedness in the natural and social world.

The students are encouraged to recreate the first iterations to replicate the Ba-ila settlement, and contemplate on questions to encourage their critical thinking such as: Why do



(a) (b)

Figure 3.8: Aerial photo of the Ba-ila settlement before 1944 (a), source: American Geographic Institute. Reproduction of the settlement as a fractal model (b).

the corrals get bigger going from the front entrance of the village to the back? Why do the buildings get bigger going from the front of the corral to the back? Can you guess what represents the line towards the back of every house? Can you guess who is the owner family of the central corral?
Some possible answers could be:

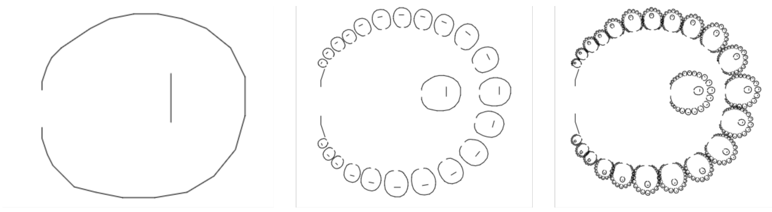
- The corrals get bigger going from the front entrance to the back in order to accommodate larger livestock, which are more valuable and usually belong to wealthier families;

- The buildings get bigger going from the front of the corral to the back in order to provide more living space for the larger families that own the larger livestock;

- The line towards the back of every house could be a representation of the family's ancestors, as many African cultures place a great emphasis on ancestor veneration and maintaining familial connections across generations;

- The central corral could belong to the most important family of the village.

The correct generator and iterations should be the following:



1^{st} iteration 2^{nd} iteration 3^{rd} iteration

3.3.3 Session No.3: Fractals in Geometry - the Sierpiński Triangle

Overview

During this session, students will delve into the fascinating world of fractals in geometry, with a particular focus on the Sierpiński Triangle. The session will kick off with an engaging game that demonstrates how fractals can arise from apparent chaos. Throughout the session, students will explore the self-similar nature of the Sierpiński Triangle, which is a fundamental characteristic of fractals. They will examine the recursive algorithm used to generate the fractal, as well as its dimensionality and unique properties. By the end of the session, students will have a much deeper appreciation for the nature and significance of fractals within the realm of geometry.

Timing

The session will take 1 hour.

Materials Required

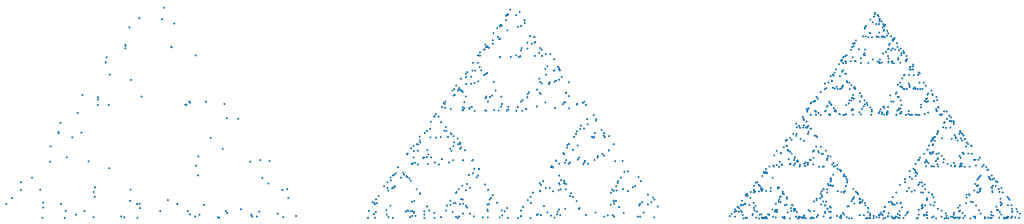
- Students: pen and paper for working.
- Facilitator: whiteboard/blackboard to present assignment and solutions to the class.

Session Outline

The facilitator should begin the session by reflecting on the previous sessions, highlighting the fact that specific rules were followed to construct fractals. In both the Koch curve and African fractal designs, the idea was to start with a generator, scale it, replace part of the first shape with the scaled copy, and repeat the process infinitely. These types of fractals are known as Iterated Function Systems or IFS Fractals.
The facilitator should suggest that students form small groups of 2-3 individuals and engage in the *Chaos Game*. The steps involved are as follows:

1. Select three points on a plane and connect them to form a triangle.
2. Randomly choose any point inside the triangle and use it as the current position.
3. Randomly select any one of the three vertex points of the triangle.
4. Move halfway from the current position to the selected vertex.
5. Plot the current position.
6. Repeat steps 3-5.

After each group has played at the chaos game for 10 minutes, the facilitator should ask them to discuss and compare their results. They should notice that despite the random selection of points and vertices, there are some areas of the drawing where there are no points. Moreover, by comparing their final plots they should notice that they will converge towards a particular shape. This shape is known as the Sierpinski triangle and is another example of an Iterated Function System (IFS) fractal. To make it more clear, the facilitator should reproduce the results of the chaos game played up to 1000 iterations or show the final outcome of the game, which is a distinct fractal shape, see Figure 3.9



(a) $n = 100$ (b) $n = 500$ (c) $n = 1000$

Figure 3.9: Python reproduction of chaos game with varying iterations.

The facilitator should now encourage the students to use their creativity and problem-solving skills to come up with an iterative method to construct the Sierpinski triangle. They should start with a black triangle and use pen and paper to draw up to the third or forth iteration of the fractal. They should come up with the following drawing:

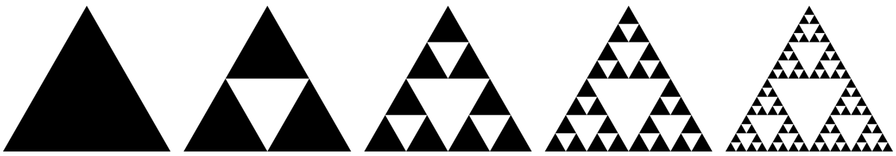


Figure 3.10: Evolution of the Sierpinski triangle

The students should explain the process they followed, which typically involves starting with a simple triangle and iteratively removing smaller triangles to construct the Sierpinski triangle. Specifically, the first step involves removing the middle triangle from the original triangle, which results in 3 black triangles surrounding a central white triangle (iteration 1). Next, the same process is repeated at a smaller scale, and the middle third of each of the 3 black triangles is removed to create the second iteration. At this stage, 9 smaller black triangles remain. The process is then repeated to generate subsequent iterations, each time removing smaller triangles at a smaller scale, until the desired level of detail is achieved.
To conclude the activity, students should be encouraged to count the number of black triangles present in each iteration of their drawing, and then try to find a pattern or rule to predict the number of triangles in future iterations without having to draw them. Through this process, they may discover that the number of black triangles follows the formula $T = 3^n$, where n is the iteration number. This formula can then be used to predict the number of black triangles in any iteration of the Sierpinski triangle without having to manually draw each iteration.

Several potential extensions can be added to this activity in order to provide a more comprehensive analysis of plane or solid geometry for students at a lower level. For example, introducing the Sierpinski Tetrahedron could help students discover that the surface area remains constant at each iteration, while the volume decreases with each iteration, approaching zero as the number of iterations goes to infinity. Engaging in this activity will enable students to apply solid geometry formulas iteratively and gain a deeper understanding of the relationship between surface area, volume, and iteration.

3.3.4 Session No.4: Fractals in Algebra - the Madelbrot Set

Overview

During this final session, participants will explore the fascinating world of iterative equations, complex numbers, and the Mandelbrot set. The session will begin with a brief overview of the concept of iterative equations, focusing on the behaviour to infinity. Participants will then dive into the basics of complex numbers and their representation on the complex plane, and how they can be used to generate complex and intricate patterns. After this introduction, the session will focus on the Mandelbrot set, which is one of the most famous examples of a fractal pattern generated by iterative equations.
Throughout the session, participants will work on various interactive exercises and guided discussions, designed to help them deepen their understanding of these fascinating concepts. By the end of the session, participants will have a much deeper appreciation for the power of iterative equations and complex numbers, as well as the intricate beauty of the Mandelbrot set, concluding the main activity about fractals.

Timing

The session will take 2 hours, depending on the level of the students. The session could also be divided in more sessions, based on the level of depth one wishes to explore with complex numbers.

Materials Required

- Students: pen and paper for working.
- Facilitator: whiteboard/blackboard to present assignment and solutions to the class.

Session Outline

To initiate the session, the facilitator should refresh the participants' understanding of the concept of iteration that was used in the previous sessions to construct fractals. In this session, the focus will be on applying iteration to an equation. The facilitator can introduce this concept by presenting the equation $Z_{new} = Z_{old}^2 + C$. This iterated equation is interpreted as the new value of a variable, Z, being equal to the old value of Z squared, plus a constant, C. This can be more formally expressed as $Z_{n+1} = Z_n^2 + C$. The counter, n, keeps track of the iteration that is being computed, where n starts at 0 and increments to produce the required image. The value of Z_{n+1} represents the next value in the cycle of iteration based on the previous value of Z.

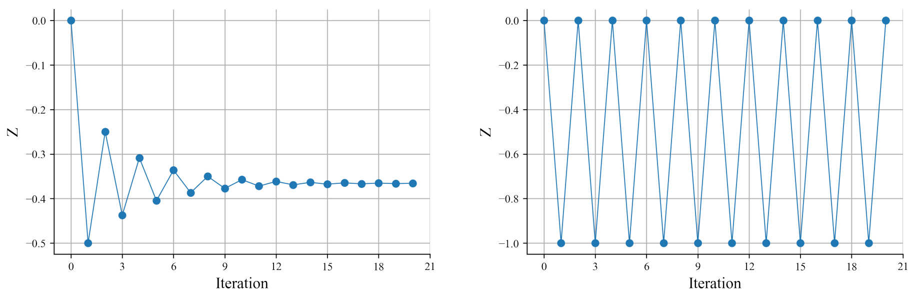
The facilitator should prompt the students to begin the exercise by setting $Z_0 = 0$ and testing the equation with different values of the constant C. For instance, starting with $C = 1$ up to $n = 4$ would be a good starting point.
The students will find the resulting sequence to be:

$$\begin{aligned} Z_1 &= Z_0^2 + C = 0 + 1 = 1 \\ Z_2 &= Z_1^2 + C = 1 + 1 = 2 \\ Z_3 &= Z_2^2 + C = 4 + 1 = 5 \\ Z_4 &= Z_3^2 + C = 25 + 1 = 26 \\ Z_5 &= Z_4^2 + C = 676 + 1 = 677 \end{aligned}$$

Afterwards, the students should be encouraged to explore this sequence, plot it on their notebooks and draw conclusions. Some observations that could be made include:

- Squaring a number tends to result in a larger value;
- As the equation is iterated, the numbers grow rapidly;
- The sequence will eventually diverge to infinity.

The facilitator will prompt the students to repeat the exercise with two different values of C: $C = -0.5$ and $C = -1$. The students will plot the sequences and observe that, in the first case, the progression eventually converges on a number between -0.35 and -0.38, while in the second case, the value of Z alternates between two fixed points. The facilitator will demonstrate that this behavior persists even as the number of iterations increases, as depicted in Figure 3.11 for $n = 20$. Through this exercise, the students will gain an understanding that some values of C cause Z_n to go to infinity, while others cause it to remain finite.



(a) $C = -0.5$ (b) $C = -1$

Figure 3.11: Progression of $Z_{n+1} = Z_n^2 + C$ with different values of C.

The facilitator may also introduce fundamental concepts of Dynamical Systems, where the path taken by the value of Z_n is called the orbit. In the case of $C = -0.5$, the fixed point -0.366 is known as an attractor, as it attracts the orbit of the equation.

The Mandelbrot set and complex numbers

The facilitator may now introduce the definition of the Mandelbrot set as the collection of all starting values C that stay finite when iterated through the equation $Z_{n+1} = Z_n^2 + C$. In order to consider all starting values, the values of C should be taken in the complex plane. Therefore, students should be introduced (or should revise, depending on the level) to complex numbers. Here is a possible way to run the session.

What is happening - The facilitator asks the students to solve different exercises:

- square of 5, 10;
- square root of 9, 64;
- square root of 12;
- ...
- we end with: what is the square root of -1?

The facilitator introduces the concept of imaginary numbers, explaining that when squared, they produce a negative result ($i^2 = -1$). The term "imaginary" is used to reflect the fact that no real number can be squared to yield -1. To illustrate the concept of imaginary numbers as multiples of the unit i, the facilitator presents examples such as $3i$, $1.04i$, and $\sqrt{2}i$. The facilitator then explains that when imaginary numbers are combined with real numbers, they form complex numbers, such as $3+1i$, $39+3i$, and $0.8-2.2i$. These complex numbers can be represented in a graphical form by plotting them on a two-dimensional X-Y plane.
After explaining how a complex plane is made, the facilitator draws a plane on the board and asks students to indicate where the previous complex number should be plotted. The facilitator then explains that complex numbers can be multiplied.
To encourage students to engage in active learning, the facilitator poses a question for them to solve: multiplying $(1 + 1i)$ by $2i$ and plotting the result on the plane. Students are then instructed to multiply the new point by i, and so on, until four points have been plotted. Finally, the facilitator asks students to observe whether they see any patterns and to connect the lines accordingly.
The purpose of this exercise is to provide students with a hands-on opportunity to explore the patterns that emerge when complex numbers are multiplied and plotted in the plane. Through this exercise, students can develop a deeper understanding of the relationships between real and imaginary numbers and the ways in which they interact when combined in complex numbers.
Finally, the facilitator can elucidate the Mandelbrot set's relation to the broader concept of fractals. Specifically, the Mandelbrot set is renowned for its complex and highly-detailed fractal structure, which manifests through self-similar patterns that arise when

zooming in on various regions of the set. This intricate fractal architecture renders the Mandelbrot set an alluring subject of mathematical inquiry and a popular avenue for artistic and exploratory pursuits. To illustrate this point, the facilitator may present a video demonstration of a zoomed-in view of any segment of the Mandelbrot set, captivating the students with the stunning beauty of mathematics.

Extensions

There are several possible extensions for this session about complex numbers and the Mandelbrot set:

- Create the Mandelbrot set using Geogebra or a similar tool: This extension can allow students to see how the set is constructed and explore its fractal structure. By inputting the iterative formula and adjusting parameters, they can generate the Mandelbrot set and experiment with zooming in on different parts of the set.
- Explore other fractals, such as the Julia set: The Julia set is related to the Mandelbrot set and can be created using similar iterative formulas. This extension can allow students to compare and contrast the properties of these sets and gain a deeper understanding of fractals and complex numbers.
- Complex arithmetic: The Mandelbrot set is generated using complex numbers. This session can be used as motivation to deepen the student's understanding of complex numbers by exploring complex arithmetic, including addition, subtraction, multiplication, and division of complex numbers.

Conclusions & Future Developments

Conclusions

In this thesis, we have explored the state of mathematics education in the context of the Sustainable Development Goals (SDGs) and investigated two case studies of maths education initiatives in Kenya and South Sudan. Our findings highlight the importance of maths education in achieving the SDGs and the potential for innovative teaching methods to enhance students' engagement and understanding. Games, technology, project-based learning, and etnomathematics are just some of the ways that can be used to enhance students' interest and engagement in these subjects.
The case study of the Kenya Math Camp demonstrated the effectiveness of a well-structured and inclusive maths camp in improving students' confidence and performance in mathematics. The positive feedback we have received from students indicates that these approaches can be powerful tools for enhancing their interest in mathematics and scientific subjects. The South Sudan case study explored the use of ethnomathematics to make mathematics more relevant and accessible to students in a post-conflict context. The fractals-based activities illustrated how local cultural practices and examples can be used to teach complex mathematical concepts in an engaging and accessible way. Moreover, as a side effect, my efforts to promote mathematics and scientific subjects have also had an impact on students' attitudes towards gender roles in STEM fields. Being a young woman in the scientific field inspired many students that I worked with, including a senior two student from Loreto Girl Secondary School who gave me a letter expressing her gratitude. In the letter, she wrote: "I have a dream of being a civil engineer. It was so amazing to me before when I heard you talk about it because I never heard a lady doing civil engineer" (see Figure A.7). This highlights the importance of having diverse role models and promoting inclusivity in STEM fields to encourage more girls and young women to pursue careers in these fields. Furthermore, my collaboration with teachers and students in South Sudan has been successful, and we are committed to continuing our support for them. By providing didactic materials and organizing math camps, we hope to help the students develop the skills and knowledge they need to succeed in their academic pursuits. Moreover, by incorporating innovative and effective teaching techniques, we can foster a more constructive and engaging learning environment in mathematics and scientific subjects. Such an environment can have a lasting impact on students' academic achievements

and future prospects.
It is crucial to recognize the constraints and obstacles faced during this research. The study was restricted to two sub-Saharan African countries, namely Kenya and South Sudan, due to limitations in time and resources. Additionally, the reliance on feedback and comments from teachers and students may lead to biases and limitations in the accuracy and completeness of the responses. Furthermore, since the initiatives were implemented over a relatively short period, evaluating their long-term impact may be difficult. Nonetheless, this thesis offers significant insights and evidence-based recommendations for enhancing mathematics education in sub-Saharan Africa.

In conclusion, this thesis underscores the crucial role of mathematics education in promoting sustainable development and the potential for innovative teaching methods to enhance students' engagement and understanding. Through the case studies of Kenya and South Sudan, we have shown how effective maths education initiatives can be implemented in different contexts and how they can contribute to a more equitable and inclusive education system.

Future Developments

As mentioned earlier, the feedback received from students who participated in the math camp in Kenya highlighted the effectiveness of using games, technology, and project-based learning, as well as incorporating etnomathematics, to enhance interest and engagement in mathematics and science subjects. Based on this feedback, there are several future developments that can be pursued.

Firstly, as of 2023, we are actively working towards organizing more math camps in other African countries, with a potential upcoming camp in The Gambia in May. The aim is to expand this experience to as many countries as possible to provide opportunities for more students to engage with mathematics in an enjoyable and meaningful way. In addition, we will continue collaborating with the teachers and students in South Sudan by providing didactic materials and support. We are also working towards organizing a math camp in South Sudan with the help of Juba University, which will provide a platform for students to engage in activities that promote interest and confidence in mathematics and science.

Furthermore, the successful use of games, technology, and project-based learning, as well as the incorporation of etnomathematics in mathematical activities, can inform and benefit mathematics education in Europe, where educators face challenges of promoting inclusion and diversity in the classroom, due to the presence of students from diverse backgrounds and cultures. In 2009, the United Nations Educational, Scientific and Cultural Organization published the "Guidelines for Inclusion in Education" to assist countries in strengthening their attention to inclusion in their education strategies and plans (UNESCO, 2009). The document introduces the concept of inclusive education and emphasizes the areas and policy actions that require particular attention to promote it. The UNESCO guidelines highlight that inclusive education is a key strategy to achieve Education for All, as education is a fundamental human right and the foundation for a more just and equal society.

Therefore, educators must focus on the central role of the person who learns and define educational projects that respond to the unique composition of the class group.

> *"Inclusive education is a process of strengthening the capacity of the education system to reach out to all learners and can thus be understood as a key strategy to achieve Education for All. As an overall principle, it should guide all education policies and practices, starting from the fact that education is a basic human right and the foundation for a more just and equal society. (UNESCO, Education for All, 2009)."*

Coming back to Italy, the Ministry of Education, University, and Research released the "National Guidelines for the Curriculum of Early Childhood and Primary Education" in 2012 (MIUR, 2012), which aimed to prepare students to face the problems of contemporary society and its rapid evolution. One of the biggest challenges in the Italian education system is the presence of children and young people with diverse cultural roots, with 828690 non-Italian citizens enrolled in schools in 2021, accounting for one in ten students in each class, according to data from the National Education Data Portal (`https://dati.istruzione.it/opendata/`). Moreover, there is a multiplicity of nationalities, with students from 192 out of 194 nations present in Italian schools. Therefore, the challenges posed by the diversity of languages, cultures, and habits must be transformed into learning opportunities.
The National Guidelines emphasize the need to prioritize the individuality and complexity of each student in all aspects, including relational, cognitive, physical, emotional, ethical, and spiritual dimensions. Thus, educational strategies and teaching methods must consider the unique characteristics of each student and aim to value the singularity and individuality of the cultural identity of each student. Teachers must design educational projects tailored to the diverse composition of the class group to promote inclusion and create an environment where students feel valued and respected.

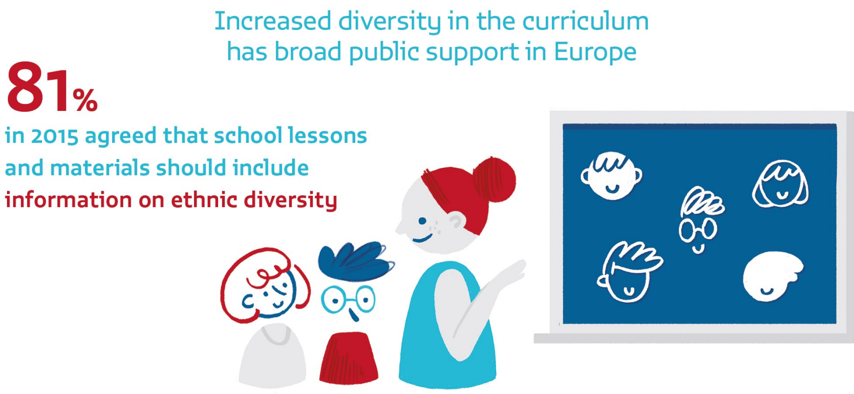


Figure 3.12: Infographic on the need for lessons and school materials that include ethnic and cultural diversity. Source: GEM Report (2018).

In this regard, etnomathematics can be an effective approach to enhance inclusivity in mathematics education, as it acknowledges the cultural diversity of learners and connects mathematical concepts with cultural practices and experiences. Etnomathematics

can contribute to promoting inclusive education by recognizing the cultural diversity of students and their ways of understanding and using mathematics. As a result, etnomathematics can create more inclusive learning environments, enhance students' interest and engagement in mathematics, and improve their overall academic performance.
In light of these benefits, as a future development of this thesis, an etnomathematics activity could be introduced in an Italian classroom to assess the effectiveness of the approach and compare it with the feedback received from the South Sudanese students. This would provide an opportunity to evaluate the impact of etnomathematics on students' engagement, motivation, and learning outcomes, and to identify best practices that can be shared with other European countries facing similar challenges.
Overall, these future developments align with the overarching goal of promoting interest and engagement in mathematics and science subjects, as well as improving the quality of mathematics education for students globally.

Chapter A

Supplementary Journals and Student Feedback

This appendix provides additional resources and materials that complement the main body of the thesis. The contents of this appendix include journals and feedback obtained from students who participated in the maths initiatives discussed in the case studies. The journals provide personal accounts of the students' experiences and reflections on their learning, while the feedback offers insights into the effectiveness of the innovative approaches used in the initiatives. These supplementary materials offer valuable perspectives and further support the main arguments and findings presented in the thesis.

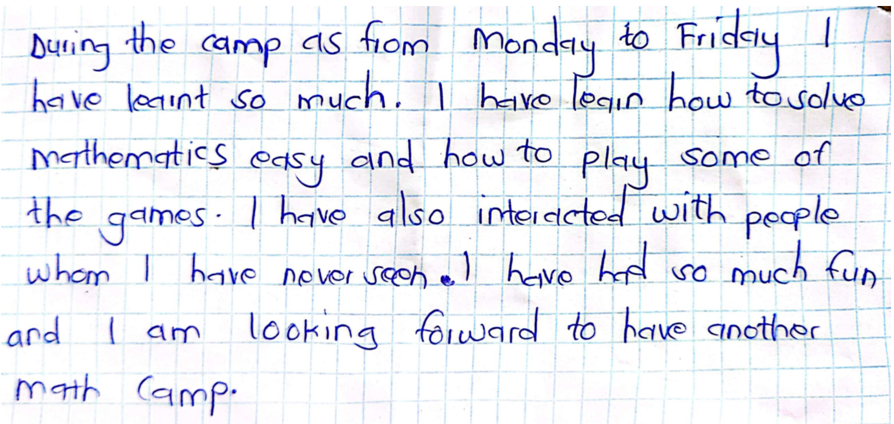
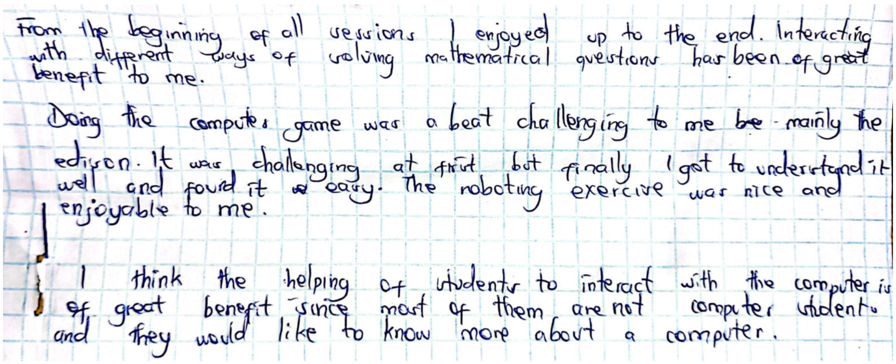

During the camp as from Monday to Friday I have learnt so much. I have learn how to solve mathematics easy and how to play some of the games. I have also interacted with people whom I have never seen. I have had so much fun and I am looking forward to have another math Camp.

Figure A.1: Journal of a student after the Math Camp

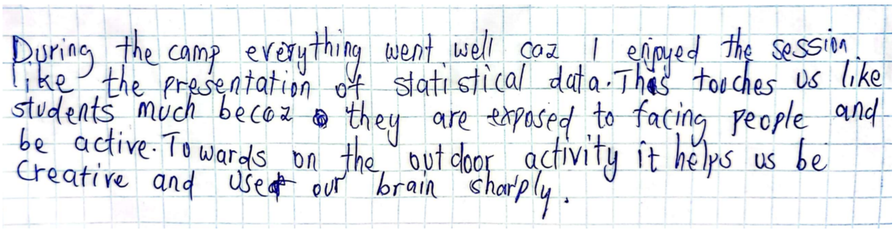


From the beginning of all sessions I enjoyed up to the end. Interacting with different ways of solving mathematical questions has been of great benefit to me.

Doing the compute game was a beat challenging to me ~~be~~ mainly the edison. It was challenging at first but finally I got to understand it well and found it ~~a~~ easy. The roboting exercise was nice and enjoyable to me.

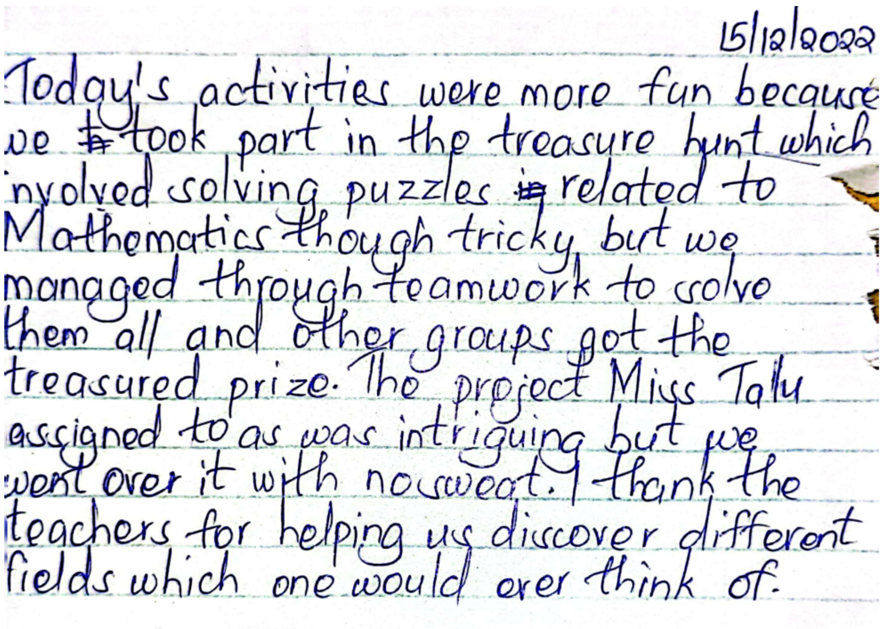
I think the helping of students to interact with the computer is ~~of~~ of great benefit since most of them are not computer students and they would like to know more about a computer.

Figure A.2: Journal of a student after the Math Camp



During the camp everything went well coz I enjoyed the session like the presentation of statistical data. This touches us like students much becoz they are exposed to facing people and be active. Towards on the outdoor activity it helps us be Creative and used our brain sharply.

Figure A.3: Journal of a student after the Math Camp



15/12/2022

Today's activities were more fun because we ~~t~~ took part in the treasure hunt which involved solving puzzles ~~is~~ related to Mathematics though tricky but we managed through teamwork to solve them all and other groups got the treasured prize. The project Miss Talu assigned to as was intriguing but we went over it with nosweat. I thank the teachers for helping us discover different fields which one would ever think of.

Figure A.4: Journal of a student after the Math Camp

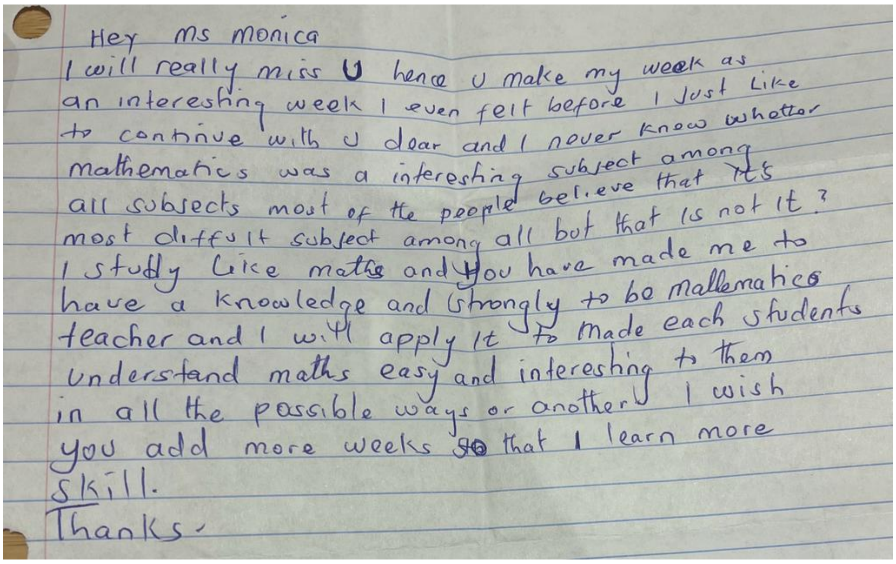

Hey Ms Monica
I will really miss U hence U make my week as
an interesting week I even felt before I Just Like
to continue with U dear and I never know whether
mathematics was a interesting subject among
all subjects most of the people believe that it's
most diffult subject among all but that is not it?
I study Like maths and You have made me to
have a knowledge and strongly to be mathematics
teacher and I will apply it to made each students
understand maths easy and interesting to them
in all the possible ways or another I wish
you add more weeks so that I learn more
skill.
Thanks.

Figure A.5: Feedback from a teacher after the teacher training

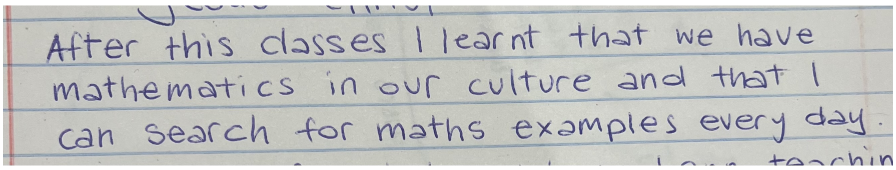
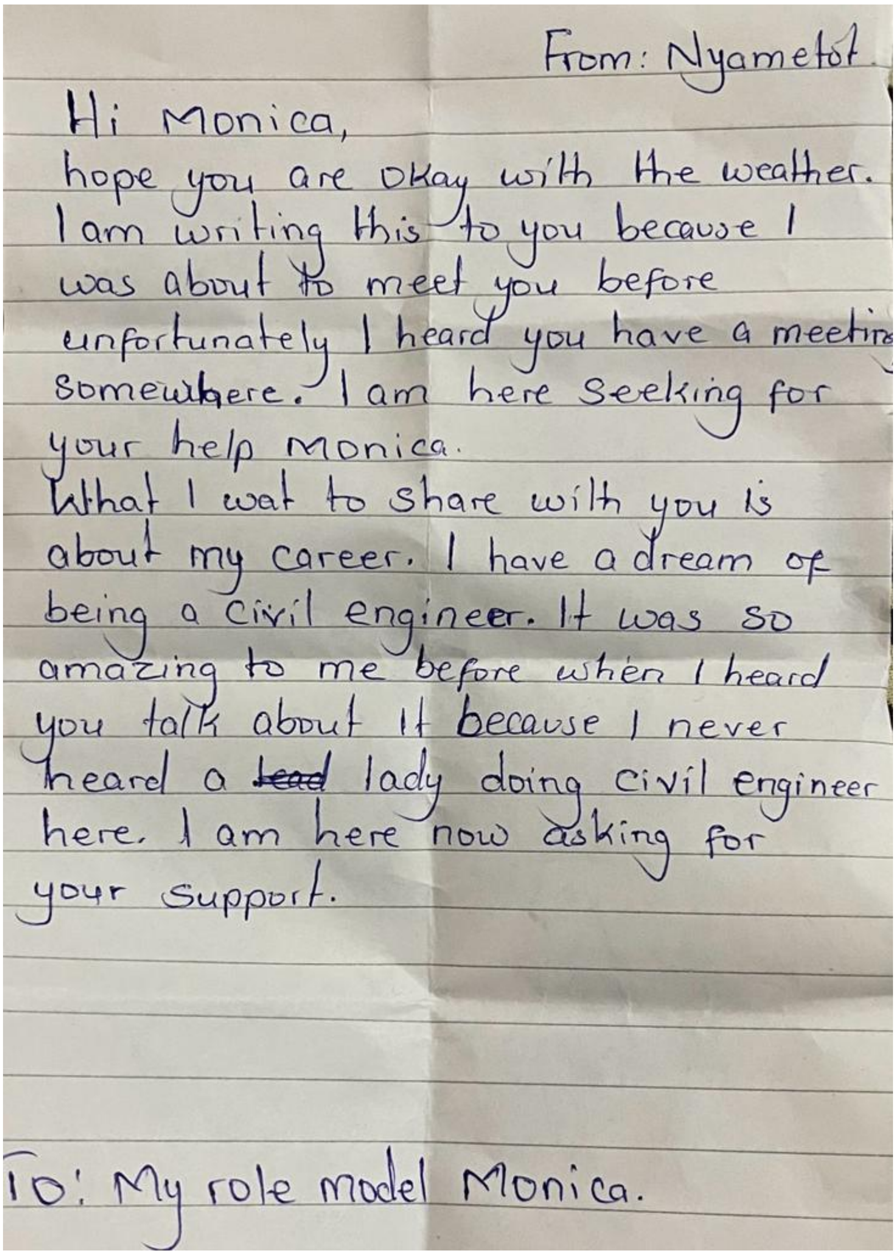

After this classes I learnt that we have
mathematics in our culture and that I
can search for maths examples every day.

Figure A.6: Comment from a student on etnomathematics activity



From: Nyametot.

Hi Monica,
hope you are Okay with the weather.
I am writing this to you because I
was about to meet you before
unfortunately I heard you have a meetin
somewhere. I am here seeking for
your help Monica.
What I wat to share with you is
about my career. I have a dream of
being a civil engineer. It was so
amazing to me before when I heard
you talk about it because I never
heard a ~~lead~~ lady doing civil engineer
here. I am here now asking for
your support.

To: My role model Monica.

Figure A.7: Letter from a student from Loreto Girls Secondary school

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